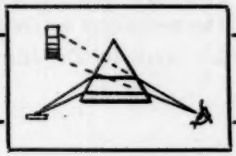
JOURNAL
of the
OPTICAL SOCIETY
OF AMERICA

VOLUME 22.
No. 11



NOVEMBER
1932

CORRESPONDING RETINAL POINTS, THE HOROPTER AND SIZE AND SHAPE OF OCULAR IMAGES*

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* Acknowledgment is made to an anonymous donor for a subvention partially to defray cost of publication of this article. Editor.

VI. EXPERIMENTAL FINDINGS

A. DETERMINATION OF THE INTERSECTS OF THE OPTICAL PROJECTIONS OF CORRESPONDING RETINAL POINTS BY THE CRITERION OF THE APPARENT FRONTAL PLANE

A great number of horopter determinations by the criterion of the frontal plane has been made by a large number of observers. In general these determinations confirm in every way the work of Fischer, both as to accuracy and the ability to repeat. As he says:[19] "The subjective certainty was slight, the objective correctness great." The improved apparatus has advanced the technique to the extent that accurate determinations can be quickly made by any intelligent observer without previous practice.

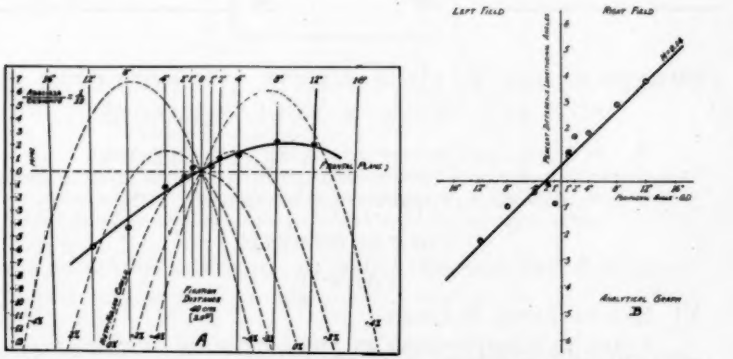


FIG. 30. *The spatial and analytic graphs for the apparent frontal plane. Data of A. Ames, Jr.*

Table 2, and Figs. A of 30, 31 and 32 show the typical results of the apparent frontal plane as determined by Mr. Ames, Dr. Ogle and Mr. Comfort.[20,21] Comparison with Fig. 23 shows that in general the curves are similar to those found in Tschermak's laboratory. The slight rotation of the curves about the point of fixation denotes a difference in the

[19] See reference 11, pp. 234–246.

[20] F. D. Comfort, a psychologist associated with Harvard University, made the measurements while visiting the laboratory in 1931.

[21] The refractive conditions of the observers at present are as follows:

(1) A. Ames, Jr.—

Rx Distance	(O.D. −0.25 sph	−0.37 Cx 165° 1% Cx 165°)
	(O.S.	−1.37 Cx 75° 1.6% sph 2.0% Cx 75°)
Rx Reading	(O.D.	−0.37 Cx 165°)
	(O.S. +0.50 sph	−1.12 Cx 75° 2.0% Cx 75°)
Static Skiametry	(OD. −0.25 sph	−0.50 Cx 160°)
	(O.S. −0.75 sph	−1.00 Cx 60°)

TABLE 2. *Tabulated data for the apparent frontal plane.*

	Left field							Right field					
(a) A. Ames, Jr. 40 cm fixation distance. Own Rx. (11 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm		−5.72	−4.37	−1.17	−0.41	0.31		0.35	0.98	1.16	2.21	1.95	
Mean dev. ±		1.9	1.0	0.5	0.3	0.1		0.2	0.3	0.4	1.1	1.9	
% value from iso-mag. curve		−2.28	−1.00	−0.45	−0.10	−0.90		1.10	1.70	1.80	2.90	3.82	
% value of mean dev. ±		0.36	0.31	0.31	0.31	0.41		0.48	0.31	0.24	0.33	0.37	
(b) K.N. Ogle 40 cm fixation distance. No Rx. (82 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−5.45	−3.20	−1.35	−0.19	−0.26	0.11		−0.14	−0.32	−1.16	−3.38	−6.56	−9.64
Mean dev. ±	1.45	0.82	0.48	0.23	0.14	0.12		0.11	0.21	0.31	0.59	0.89	1.62
% value from iso-mag. curve	−3.73	−2.75	−1.84	−1.02	−0.85	−0.60		0.00	0.02	0.47	1.29	2.15	3.25
% value of mean dev. ±	0.20	0.16	0.14	0.13	0.16	0.34		0.31	0.25	0.18	0.18	0.17	0.22
(c) F.D. Comfort. 75.6 cm fixation distance. Own Rx. (5 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−20.0	−13.1	−8.9	−2.0	−1.7	−0.5		0.4	−1.2	−3.8	−10.9	−18.2	−23.1
Mean dev. ±	2.5	3.9	1.5	1.7	1.4	0.7		0.5	0.4	1.5	3.3	4.9	10.9
% value from iso-mag. curve	−1.63	−1.07	−0.47	−0.27	0.26	0.20		−0.14	−0.12	−0.03	0.30	0.83	1.53
% value of mean dev. ±	0.12	0.21	0.13	0.25	0.48	0.43		0.25	0.10	0.22	0.30	0.27	0.48

relative size of the ocular images. For example, in Mr. Ames' case the ocular image in the horizontal meridian in the left eye is slightly larger than that of the right. The departure of the curve from the Vieth-Müller circle indicates an asymmetry in the images and has been designated the Hering-Hillebrand deviation value.[22] Figs. B of 30, 31 and 32, give the same data plotted by a transformation according to the method described above. In this type of graph the magnitude of the difference in size of the ocular images is given by the distance above or below the horizontal axis, where the curves intersect the vertical axis.

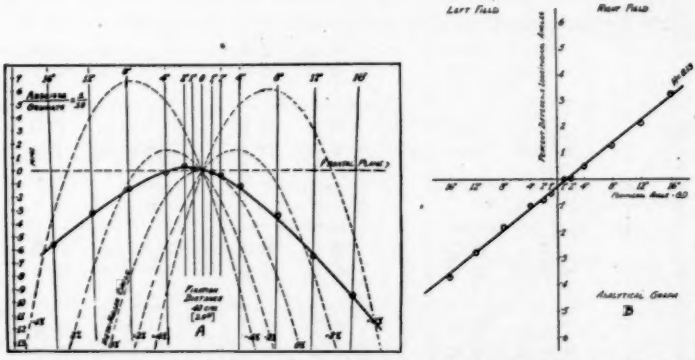


FIG. 31. *The spatial and analytic graphs for the apparent frontal plane. Data of K. N. Ogle.*

Dissociated phoria with Rx at 20 ft. = 1.75△ eso; at 40 cm with minus cylinders 5△ eso. Subjective determinations with clinical machine at 20 ft. show that the conjugate point for the O.D. is +0.55 D.; for the O.S. +0.62 D. At 40 cm the conjugate point for the O.D. is +2.1° D., for the O.S. +2.3° D. The cyclo-declination, horizontal, at 20 ft. is +1.00°; at 40 cm +1.7°. The vertical declination at 20 ft. is +1.5°; at 40 cm +0.9°. The P.D. = 68 mm.

The "size" lenses correct differences in the relative size of Mr. Ames' ocular images.

The above Rx is that which Mr. Ames wore during the time that the data given in this paper were made. However, a change has occurred in his eyes since then, probably associated with presbyopia, and hence that prescription could be modified, as is evident from the above findings. Not only has Mr. Ames' refractive condition altered, but his phoria has changed from exophoria to esophoria.

(2) K. N. Ogle—no Rx worn. Static skiametry at 20 ft. +0.50 D. sph. O.U. Dissociated phoria: horizontal, at 20 ft. 2△ eso; at 40 cm 11△ eso; vertical, none. Subjective determinations with clinical machine show, at 20 ft., the conjugate point for the O.D. to be +0.15 D., for the O.S. +0.26 D. At 40 cm the conjugate point is O.D. +2.1 D., O.S. +2.3 D. Cyclo-declination at 20 ft., horizontal +0.5°; vertical +2.0°. P.D. = 64 mm.

(3) F. D. Comfort—Rx (O.D. +0.25 sph +0.25 Cx 90°)
(O.S. +0.37 Cx 90°)
Static skiametry (O.D. +0.25 sph +0.25 Cx 90°)
(O.S. +0.67 Cx 90°)
Dissociated phoria:
20 ft. orthophoria; at 40 cm 9△ exophoria. P.D. = 64 mm.

The curve for Mr. Ames' data shows the ocular image in the horizontal meridian of his O.S. to be 0.6 percent larger than that of his O.D. In Dr. Ogle's case the O.D. image is a fraction of a percent larger than the O.S. image. The slope of the curve in the graph gives a quantitative measure of the Hering-Hillebrand deviation. For Mr. Ames and Dr. Ogle these slopes are 0.153 and 0.122, respectively. Further examples illustrating the horopter as determined by the apparent frontal plane may be found in the section on variations in the horopter with fixation distance.

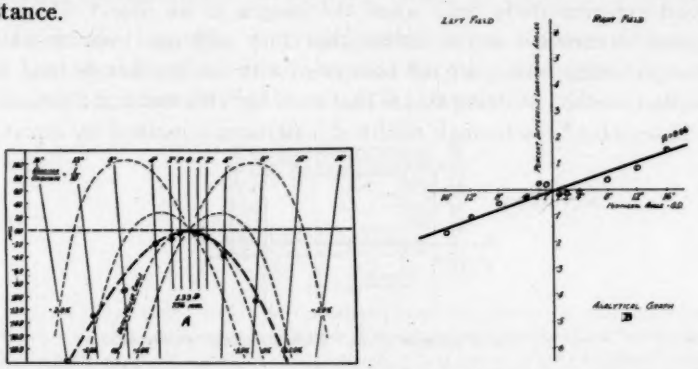


FIG. 32. *The spatial and analytic graphs of the apparent frontal plane. Data of F. D. Comfort.*

The accuracy and consistency of the findings are indicated by the mean deviations of the settings from their mean as shown in the tables.[10] From a series of five settings with the improved apparatus, it is possible to determine quite accurately the position of the apparent frontal plane of any normal individual.

A study of the trace of the horopter so determined, affected by, first, change in the condition of phoria; second, change of fixation distance and symmetrical convergence; third, change of size and shape of the dioptric image produced by lenses, will be presented in later sections.

B. DETERMINATION OF THE INTERSECTS OF THE PROJECTIONS OF CORRESPONDING RETINAL POINTS BY THE CRITERION OF THE CONSTANCY OF VISUAL DIRECTION

The grid-nonius method

Tschermak considers the criterion of the constancy of apparent separation of two pairs of corresponding points the most reliable for the

[10] The Hering-Hillebrand horopter deviation values used throughout this and other papers from this laboratory have greater significance than the mere graphical deviation from the Vieth-Müller circle. Moreover, the graphical deviation may be misleading as to the real magnitude of the deviation constant.

determination of the horopter. If two objects appear to have the same directions as seen by each eye, their images must fall on corresponding retinal points. The intersects of the projections of a pair of corresponding points on the horizontal meridian, can be found by fixating the eyes and adjusting a lateral object so that the apparent angular separation between it and the fixation object will appear the same to both eyes. Since true visual direction of an object cannot be made evident so long as fusion exists, the intersect of corresponding retinal points can be found experimentally only when the images of an object which are formed in each eye are so unlike that they will not fuse, or which, through configuration, are not associated with images that do fuse. The simplest method of doing this is that used by Volkmann and described in Helmholtz.[23] Tschermak modified Volkmann's method by equating

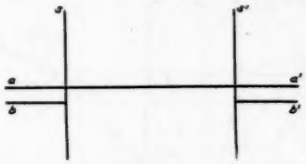


FIG. 33. *The illustration of the Volkmann nonius arrangement.*

the apparent direction in one eye, not against that in the other, but against the apparent direction as seen binocularly. His description of this method, which he calls the binocular nonius method, is as follows:

[23] Helmholtz's *Physiological Optics*, Trans. Ed. James P. C. Southall, *3*, pp. 413–415. Helmholz first described the method of determining corresponding retinal points on the vertical meridian as follows: "Careful experiments on this subject were carried out by Volkmann also. Two rectangular crosses composed of two horizontal lines (Fig. 33) a, a', and two vertical lines, s, s' were so adjusted that the center of each was exactly opposite one eye, the distance between the two being, therefore, the same as the observer's interpupillary distance. Extending outwards from the vertical lines, two other horizontal lines (b, b') were drawn below a and a'. The line b was stationary, but the other line b' could be raised or lowered. Adjusting the visual axes so that they are parallel, the observer gazes at the centers of the two crosses until they appear to be fused; and then he raises or lowers the horizontal line b' until it seems to be the exact continuation of the fixed line b on the other visual globe. In Volkmann's experiments the distance of b below a was 5.50 mm. The average of thirty trials gave 5.51 mm for the distance of b' below a', where the adjustable line was opposite the right eye; and 5.47 mm, when this line was opposite the left eye. The distance of the two crosses from the eyes was 300 mm, and so the difference between the two values to be compared here is below the limit of the minimum perceptible interval." Helmholtz then goes on to describe similar determinations in the horizontal meridian, saying, "A series of experiments was carried out by Volkmann to determine this fact (position of corresponding points in the horizontal meridian); the method being similar to that just described, except that instead of a pair of horizontal lines, one of which was stationary and the other adjustable, he had in this case a pair of vertical lines. Each of these vertical lines was to the right of the vertical line of the cross, but one of them was

"Finally, the most reliable criterion of correspondence is shown by the criterion of the constancy of visual direction, i.e., the maintenance of the visual direction of a plumb in the comparison of binocular and monocular observation, in a simultaneous presentation of a binocular and of a monocular space. For this purpose merely cover a plumb in the median distance (for one eye but not for the other), with a small card whose brightness is such that its impression just fuses with that of the background (binocular nonius method according to Tschermak; see Fig. 34).[18] If the position of the plumb, which is apparently divided into three spaces, i.e., into an upper and lower binocular and a median longitudinal distance, is just so that all three spaces are exactly in one row, then both eyes are struck by accurately correspondent longitudinal rows. The setting is made (presupposing individual qualification), with great precision, hence with a very small error."

Tschermak's reason for determining the relation between monocular and binocular apparent space was that he considered that binocular visual direction was different in nature from monocular visual direction.

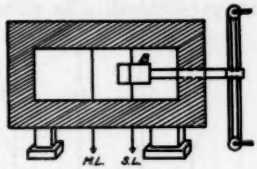


FIG. 34. *The apparatus used in the Tschermak-Fischer binocular nonius method. Device for determining the empirical horopter according to Tschermak's nonius method. R = Frame covered with wool black paper, M.L. = Fixed median plumb; S.L. = Lateral plumb; B = Small sheet of paper covering the S.L. for the O.D. (according to F. P. Fischer).*

These studies, as will be shown later, would indicate that while binocular visual direction may appear different from monocular visual direction, under certain conditions it is based on and can be derived from the latter.

Fischer used Tschermak's binocular nonius method to determine which of the numerous different horopters he had found, i.e., white and black, red and blue, permanent and momentary, were true horopters. Concerning the method he states, "It is founded on the comprehension of correspondence as a fixed elementary reciprocity of visual direction of definite corresponding points, which never breaks down in a person with normal binocular vision, hence never permits double vision with such points." He found that the black, the blue and the red horopters

above, and the other below the horizontal line. The one above was the stationary line, and it was situated 5.20 mm from the vertical line of the cross. When the latter line was opposite the right eye, its distance from the vertical line of the cross was found to be 5.24 mm, as the average of thirty settings. With the left eye, the average of the same number of settings gave 5.21 mm. Here again, therefore, the differences to be compared are less than the size of the minimum perceptible interval. Evidently, these measurements too were made by Volkmann with very great accuracy." See also note by v. Kries, top of page 489.

pass the binocular nonius test and are therefore true simple horopters. The white horopter does not, nor does the momentary horopter.

The mechanical means in this method of moving the peripherally seen objects laterally to one another is a great improvement on that used by Volkmann.

A number of horopter determinations have been made in this laboratory by this method with most interesting results. In general it was not found possible to make the method perform exactly as Fischer described it. It is gathered that for him the monocularly seen center section moved laterally relative to the upper and lower binocularly fused sections as the thread was moved forward or backward. Dr. Ogle got this effect only with the 4° and at times with the 2° wires in his left field. Mr. Ames could not get it at all. The effect as seen by Dr. Ogle is

FIG. 35. *The appearance of the horopter wires to K. N. Ogle and A. Ames, Jr., when using the Tschermak-Fischer binocular nonius method.*

shown in Fig. 35, where at a certain position of a given wire only one vertical line could be seen. See *a*. As the wire was moved the appearance became that shown in either *b* or *c*. With Mr. Ames the appearance as illustrated at *d* or *e* resulted, without question, from an excyclotropia. In general, with both Dr. Ogle and Mr. Ames when the wires appeared as lines, the settings equated the space values seen by one eye against those seen by the other and not against binocularly seen space value. In view of this fact the settings were not in accordance with Tschermak's binocular nonius method.

However, relatively definite horopter determinations can be made in this way. Examples of this are given in Table 3 and Fig. 36. In passing it should be noted that the horopter so determined is displaced relative to the position of the apparent frontal plane. Reasons for this will be given later.

Since the binocular factor was not made use of, the method was modified to prevent that factor from operating at all and at the same time to improve it as a means of equating the direction seen by one eye against that seen by the other. This, as has been described, was accomplished

TABLE 3. *The data obtained by K.N. Ogle using the Tschermak-Fischer binocular nonius method. No Rx. Fixation distance 40 cm (5 readings).*

	Left field						Right field				
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−15.3	−9.9	−4.7	−3.0	−2.1		−2.0	−2.4	−3.7	−5.3	−9.3
Mean dev. ±	2.3	1.8	2.0	0.5	0.5		0.5	0.9	1.4	2.6	2.9
% value from iso-mag. curve	0.39	0.60	1.61	2.90	4.30		−4.40	−2.10	−1.00	0.72	1.63
% value of mean dev. ±	0.45	0.55	1.15	0.60	1.10		1.15	1.00	0.80	0.80	0.55

by the grid, which gives a very sensitive nonius method. Due to the fact that no part of any wire is fused binocularly, not only do the

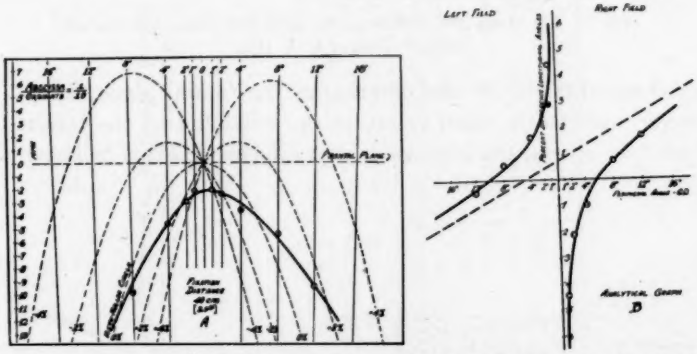


FIG. 36. *The spatial and analytic graphs for the data of K. N. Ogle obtained with the Tschermak-Fischer binocular nonius method.*

dashed upper parts of the lines move freely past the solid lower parts, but there is no suggestion for localizing the distance of the wires resulting from binocular depth perception. Moreover the difference in appearance between the dashed and solid lines makes it possible to clearly see their relative position out at least to 16°. A further advantage is that the region in which space values are being determined is limited more nearly to the visual plane. This method has been named the "grid-nonius." It evolved from the use of Tschermak's method, although it does not make use of his binocular factor. The advantages of the grid-nonius method will become apparent later in the paper.

A large number of horopter determinations have been made by this

method. In all, Dr. Ogle has taken around 600, Mr. Ames about 200, not to mention numerous others by assistants and clinical patients. The mean deviations of the settings by this method are considerably greater

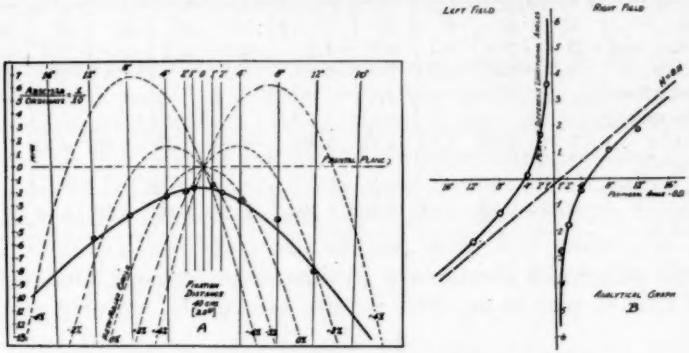


FIG. 37. *The spatial and analytic graphs of the grid-nonius longitudinal horopter. Data of K. N. Ogle.*

than those obtained by the criterion of the frontal plane. They are, however, sufficiently small to insure the reliability of the results obtained.[10] In general the settings are not difficult and it is believed that

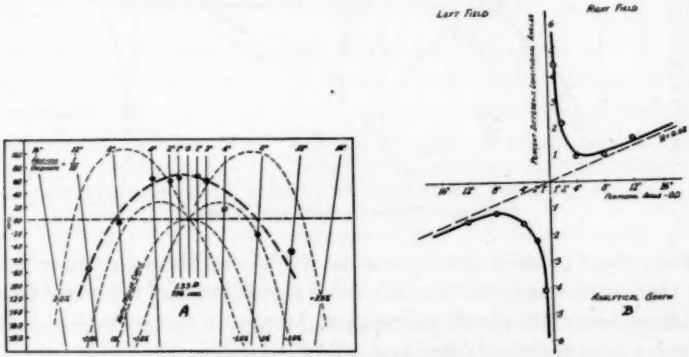


FIG. 38. *The spatial and analytic graphs of the grid-nonius longitudinal horopter. Data of F. D. Comfort.*

from a set of five by any normally intelligent person, the trace of the longitudinal horopter can be accurately determined. Horopters so determined vary quite consistently with the variation of the dioptric image as do those determined by the criterion of the frontal plane. In form they are somewhat similar to the latter in their peripheral regions,

TABLE 4. *Grid-nonius longitudinal horopter data.*

	Left field						Right field				
(a) K.N. Ogle. No Rx 40 cm fixation distance eso-fixation disparity (15 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−5.5	−3.8	−2.3	−1.9	−1.7		−1.4	−2.1	−2.7	−4.1	−8.1
Mean dev. ±	2.8	1.6	0.9	0.6	0.4		0.4	0.6	0.8	1.8	4.0
% value from iso-mag. curve	−2.38	−1.28	0.18	1.7	3.6		−2.85	−1.83	−0.45	1.07	1.88
% value of mean dev. ±	0.39	0.47	0.53	0.68	1.0		1.0	0.68	0.50	0.52	0.56
(b) F.D. Comfort. Own Rx 75.6 cm fixation distance exo-fixation disparity (5 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−7.40	0.50	6.10	5.80	6.20		6.50	5.80	1.60	−2.10	−4.90
Mean dev.±	5.75	6.61	3.61	1.05	2.49		2.70	3.97	5.6	5.05	5.53
% value from iso-mag. curve	−1.40	−1.20	−1.60	−2.20	−4.10		4.40	2.20	0.90	1.00	1.60
% value of mean dev.±	0.31	0.60	0.53	0.36	1.55		1.68	1.35	0.83	0.46	0.30

but they differ markedly from them near the point of fixation. Table 4 gives the experimental data for typical examples of the grid-nonius horopter for Dr. Ogle and Mr. Comfort. Figs. 37 and 38 show these data graphed in the old and new way.

It will be seen from the spatial graphs in these figures that the horopter curves drawn through the empirical points do not pass through the point of fixation. This behavior is characteristic of practically all horopters determined by the grid-nonius method. It was also found in those determined by Tschermak's binocular nonius method, see Fig. 36. Its importance cannot be overstressed. Its explanation is found in a

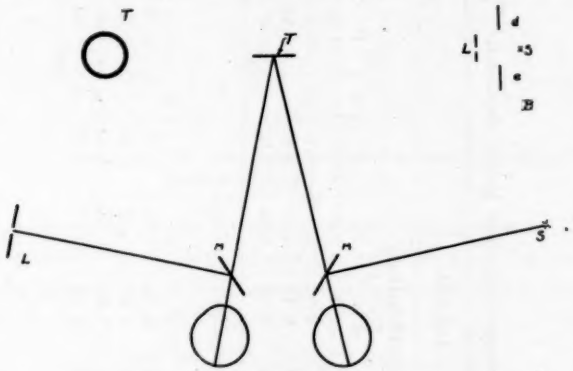


FIG. 39. *A diagram of the haploscopic apparatus for demonstrating the phenomenon of fixation disparity in the foveal region.*

phenomenon studied in this laboratory and first described[1] as "retinal slip" and now called "fixation disparity." As was pointed out, this phenomenon means that the fixation intersect does not coincide with an intersect of the projections of corresponding retinal points. This phenomenon was mentioned by Lau and Hofmann in papers dealing with binocular depth localization. Lau,[24] explaining a disparity he found at the fovea says, page 15, "It appears from this that in normal life we probably always assume an eye position which deviates from the mathematical convergence, and indeed the empirical line of regard of the two eyes is always directed to a point behind the fixated point." Hofmann[25] dealing with the same phenomenon recognizes its correlation with the separation of accommodation and convergence. In the paper

[24] Ernst Lau, Z. f. Sinnesphysiol., *53*, pp. 1–35; 1921.
[25] F. B. Hofmann, Gr.-S. Handb.; 2nd Ed. *3*, p. 428; 1920–1925.

Ocular Measurements[26] from this laboratory, the same phenomenon is described under the name "retinal slip" and there it was definitely correlated with phoria. All work in this laboratory involving corresponding retinal points confirms this conclusion and definitely shows that with phoria there is a misdirection of the fixation visual lines, that is, the images of the fixation point at the fovea fall on disparate retinal points. The direction and magnitude of this disparity varies within limits, with the nature and magnitude of the phoria, whether natural or induced by prisms or lenses.[27]

The horopter takes characteristic positions behind or in front of the point of fixation depending upon whether the "fixation disparity" is exophoric or esophoric, respectively, for that fixation distance. In Mr. Comfort's case for instance, due to his exophoria his eyes are slightly diverged relative to the fixation point. As a result his horopter lies behind his fixation point. In Dr. Ogle's case, since he is esophoric, his eyes are slightly more converged relative to his fixation point, and his horopter lies nearer. In all cases the fixation point is not imaged on corresponding retinal points. It is seen as single, however, because it is imaged within corresponding fusional areas.

If this characteristic positioning of the horopter relative to the fixation intersect is in fact correlated with phoria it should be present with induced as well as with natural phoria. This was found to be the case. Horopter determinations were made by Messrs. Ames, Ogle and Comfort after a different phoria and therefore fixation disparity condition

[26] Ames and Gliddon, *Ocular Measurements*, reference 1.

[27] Ames, Gliddon and Ogle, *Size and Shape of Ocular Images*. In the empirical grid-nonius horopters found here this disparity was determined only outside of 1° of the axis of vision. That it extends across the axis of vision is made clear as follows. Through the half-silvered mirrors of a haploscope, Fig. 39, a fusion target as shown at *T* is seen by both eyes. The left eye also sees a vertical broken line by reflection from the half-silvered surface on the left arm of the haploscope shown at *L*. The right eye sees a small point of light, shown at *s* by reflection from the half-silvered surface on the right arm of the haploscope. The two haploscopic images are carefully aligned by a telescope to coincide with the center of *T*. An observer with a phoria and fixation disparity looking into the haploscope will see the broken line and light separated whether he looks directly at them or anywhere on the target. The card carrying the broken line *L* is adjustable and the amount of displacement necessary to align the light and lines can thereby be accurately measured. The angular displacement so determined checks numerically very closely with the angular difference in position of the fixation intersect and the point directly in line with it through which the extended horopter cuts. For this displacement to be apparent it is necessary first, that objects be dissimilar so they will not fuse; second, that they be not closely associated by configuration to images which do fuse. That is the reason for the particular shape of the fixation object used in Fig. 39. If lines as shown in Fig. 39B are used for fixation object no slip can be seen as both the light *S* and the line *L* through configuration are interpreted as being in a continuation of *d* and *e*.

had been induced by lenses and prisms. Table 5 and Fig. 40, giving the data of Mr. Ogle, show typical results when an exo-fixation disparity is induced by power lenses. Compare Fig. 40 with Fig. 37.

TABLE V. *The grid-nonius longitudinal horopter data for K.N. Ogle in which an exo-fixation disparity was induced by the wearing of +2.13 Diopter spheres O.U. 40 cm fixation distance. (10 readings).*

	Left field						Right field				
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−5.35	−0.71	−0.67	0.23	0.31		1.47	1.92	2.45	2.62	1.19
Mean dev.±	1.89	1.55	1.06	0.67	0.39		0.42	0.50	0.53	1.76	2.55
% value from iso-mag. curve	−2.40	−2.08	−0.75	−0.80	−1.05		3.60	2.78	2.50	2.98	3.58
% value of mean dev.±	0.36	0.45	0.62	0.81	0.88		0.95	0.60	0.31	0.51	0.68

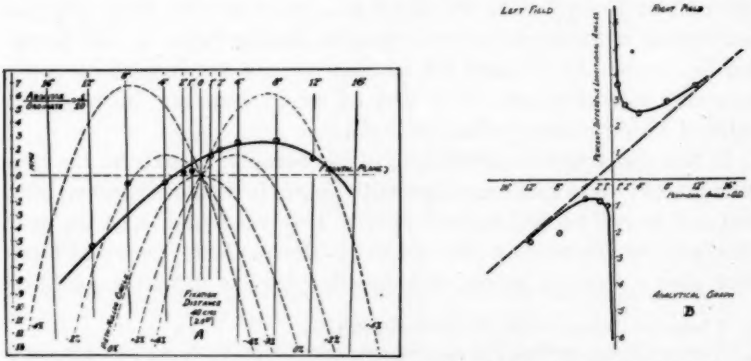


FIG. 40. *The spatial and analytic graphs for the grid-nonius longitudinal horopter. Data of K. N. Ogle when an exo-fixation disparity was induced by the use of 2.12 diopter positive power lenses before the eyes.*

What appears to be a co-phenomenon is found in Panum's limiting case[28] which apparently can be explained by fixation disparity. Panum's limiting case (the "Wheatstone-Panum limiting case") as described by Hoefer and Tschermak[29] is that "the lateral distance of two equal objects, for instance of two plumb-lines the anterior of which completely screens the posterior for one eye is perceived in binocular vision considerably smaller than in monocular vision by the eye by which the object involved is alone seen." The experimental set-up that has been

[28] Reference 25, p. 429.

[29] P. Hoefer, Pflügers Arch., *115*, p. 483; 1906. See also Tschermak, Bethe's Handb., *7*, Part 2, p. 942; 1930.

used to demonstrate this phenomenon is shown schematically in Fig. 41 where the eyes are fixated on a vertical wire F and where P and Q are vertical wires, the first always being completely hidden to the left eye by F while the second should be adjusted so as to lie on the horopter. In the experiment if F, P and Q are to appear equally spaced, it will be found that in general a different adjustment of P (or Q) will have to be made in binocular vision than in monocular vision (the O.S. being covered).

The phenomenon of Panum's limiting case can be demonstrated by a modified method which avoids the possibility of an error existing in

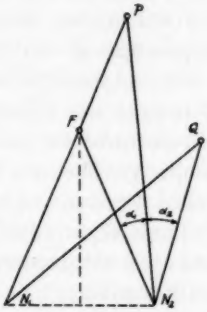


FIG. 41. *Schematic diagram of the apparatus used to demonstrate the phenomenon of Panum's limiting case.*

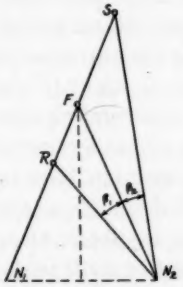


FIG. 42. *Schematic diagram of the apparatus used to demonstrate the phenomenon of Panum's limiting case which avoids any possible error in the adjustment of one of the threads on the horopter*

the position of the wire Q on the horopter. The experimental set-up is shown schematically in Fig. 42. The eyes are fixated on a vertical wire at F, which carries a small fusional target. Mounted on a slide which is movable along the line of sight of the left eye independent of F, are two vertical wires R and S, that have a constant separation. The left eye always sees only one vertical black rod, while the right sees three. For the experiment the slide carrying R and S is adjusted until the three wires appear equally spaced, first in monocular and then in binocular vision.

The empirical results of previous experimenters show that the apparent separation of the objects when seen monocularly is in general greater than when seen binocularly. According to Hoefer, "The image of the fixed plumb-line which screens the second plumb-line from one eye affects both its corresponding image in the other eye (vertical fused

image) and the disparate image of the screened plumb-line and in this manner determines its apparent distance and its vertical perception. The meridian of the eye stimulated by the fixed plumb-line according to A. Tschermak's conception thus fulfills two functions: plani- and stereo-function." While exhaustive determinations have not yet been completed in this laboratory, the results of several experimenters using both methods described above, show convincingly that the phenomenon is due to fixation disparity, both in direction and order of magnitude with normal and induced fixation disparity. The preliminary experiments were made with normal fixation disparity and with abnormal disparity induced by lenses or prisms. In every case the change in adjustment from monocular to binocular vision was in that direction indicated by the subjective values of the nearest pair of corresponding points for the particular fixation object. Thus in the method illustrated in Fig. 38, in which Q remains fixed on the horopter, the adjustment of P from monocular to binocular vision must be toward F for eso-fixation disparity and away from F for exo-fixation disparity. The data of previous experimenters have in general followed the latter case, as might be anticipated, since in an average group of individuals 80 percent to 90 percent show exophoria at near vision. The same type of adjustment of S and R must be made in the method illustrated in Fig. 42.

It should be noted that while the position of the horopter as determined by the criterion of constancy of visual direction using the grid-nonius method varies with the nature and amount of phoria and fixation disparity,[30] the position of the horopter as determined by the criterion of the frontal plane remains almost constant, showing only a slight variation near the fixation point. The above determinations show that the trace of the horopter as determined by the criterion of the apparent frontal plane is not necessarily the trace of the intersects of the projections of corresponding retinal points, i.e., the true horopter. Why this phenomenon is not apparent in horopter determination by the criterion of the frontal plane may be as follows. With the grid-nonius method the position of the fixation wires in space relative to the horopter is determined by the direction and amount of disparity of their images relative to corresponding retinal points in the two eyes. If objects which form similar images on the peripheral retina of both eyes are used, their images will fuse, they will have the same disparity and take

[30] That greater percent differences exist between the apparent frontal plane and the grid-nonius horopter near the center than in the periphery is evident from inspection of the manner in which the iso-magnification curves separate toward the periphery.

the same apparent position in space relative to the true horopter that the fixation wire does in the grid-nonius method.

Fischer, using the binocular nonius method checked the position of the black horopter that he found by the frontal plane criterion. There are three possible reasons why Fischer failed to find a difference. First, while the theory of Tschermak's binocular nonius method is sound, there is a question as to whether the particular means employed by Fischer would show what was expected. As was pointed out, where fixation disparity exists it can be made evident only by the use of contours which do not fuse and *which are so positioned relative to other contours that do fuse, that they are not affected by configuration.* The success of the device used by Fischer depends upon the appearance of a lateral displacement of the monocularly seen central section of the thread *relative to the ends seen binocularly.* Now the eye that sees this central section of the thread also sees the sections of the thread that lie above and below it. The whole appears as one continuous thread. A lateral displacement of the central portion may be prevented by the configuration. It has been found that it is impossible to make fixation disparity apparent with such an arrangement, even when it is known to exist.[31] Second, Fischer's horopter determinations did not include data for peripheral angles, less than about 5.7°. As the above findings indicate and mathematical analysis[16] shows, the displacement of the horopter from the frontal plane due to fixation disparity is most apparent near the fixation point, becoming less evident with an increase of peripheral angle. At larger angles it might easily be overlooked. Third, Fischer may not have had any fixation disparity. The horopter determined by the criterion of the frontal plane may conform to that determined by the criterion of constancy of visual direction if there is no fixation disparity. Fischer states that he has a slight exophoria as determined with dissociated eyes. This determination was probably for distant vision. It is possible that he had no phoria at the distance at which he made his horopter determinations.

The two other criteria mentioned by Tschermak by which the true horopter can be determined are: the criterion of maximal differential sensibility and that of median single vision. The former criterion was used by Tschermak and Kiribuchi.[32] They found the greatest sensitivity to depth differences to be in the vicinity of the apparent frontal plane. The precision of such measurements probably precludes the possibility

[31] See also Spearman, Trans. Ophth. Soc. U. K., *41*, pp. 91–97; 1921.
[32] Pflügers Arch., *81*, pp. 328–348; 1900.

of detecting fixation disparity. There was no reference to the phoria condition of Kiribuchi. No determinations have been made in this laboratory on the basis of this criterion. Determinations have been made, however, on the criterion of median single vision,[10] i.e., the middle of Panum's longitudinal area. See Table 6 and Figs. 43 and 44. The first shows the data of Mr. Ames under normal conditions with his own

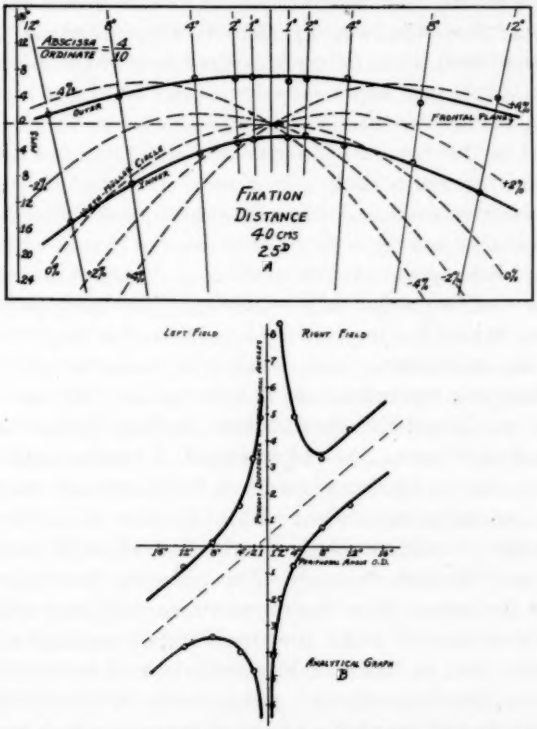


FIG. 43. *The spatial and analytic graphs of the region of binocular single vision which describes the longitudinal magnitudes of the fusional (Panum's) areas. Data of A. Ames, Jr.*

Rx made when he had an exo-fixation disparity. It will be noted from the spatial graph that the entire region of binocular single vision is displaced in the direction of his fixation disparity. These data thus substantiate the results of the grid-nonius method. The second shows the data of Dr. Ogle when a 3 percent magnifying zero power lens is placed before the right eye. Here the region of single vision rotates about the point of fixation as does the grid-nonius horopter when a

TABLE 6. *The experimental data of the region of binocular single vision in the determination of the longitudinal magnitudes of the fusional areas (Panum's areas).*

	Left field						Right field				
(a) A. Ames, Jr., Own Rx. 40 cm fixation distance. The outer limits. (6 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	1.3	4.0	6.9	6.6	7.1		6.3	6.5	6.5	3.1	3.7
Mean dev.±	1.8	2.3	1.7	1.3	0.9		0.8	1.5	1.6	2.6	2.3
% value from iso-mag. curve	−3.80	−3.45	−3.98	−4.70	−14.9		15.6	8.40	4.90	3.30	4.40
% value of mean dev.±	0.34	0.67	1.00	1.57	2.03		1.81	1.82	0.94	0.75	0.44
The inner limits (6 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−15.0	−9.0	−4.8	−3.0	−2.2		−1.8	−2.2	−3.4	−5.9	−10.6
Mean dev.±	1.3	1.6	1.0	0.8	0.5		0.7	1.0	1.3	1.9	3.1
% value from iso-mag. curve	−0.75	0.20	1.60	3.27	4.85		−4.10	−2.00	−0.75	0.70	1.63
% value of mean dev.±	0.25	0.46	0.59	0.97	1.13		1.58	1.21	0.77	0.55	0.59
(b) K.N. Ogle. 3% magnifier before O.D. 40 cm fixation distance. The outer limits (5 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	14.5	10.9	7.0	5.3	3.9		1.2	−0.6	−4.1	−11.0	−20.2
Mean dev.±	2.4	2.1	1.0	0.6	0.7		0.6	0.9	2.1	2.7	4.0
% value from iso-mag. curve	−6.10	−5.28	−4.95	−6.20	−9.65		2.90	−0.08	−1.15	−0.80	−0.28
% value of mean dev.±	0.46	0.61	0.59	0.73	1.58		1.36	1.09	1.24	0.78	0.76
The inner limits (6 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	0.4	1.2	2.1	−0.06	−0.5		−3.2	−5.7	−10.2	−21.2	−33.4
Mean dev.±	2.5	2.1	1.0	0.8	0.5		0.4	0.7	0.7	1.0	1.5
% value from iso-mag. curve	−3.65	−2.70	−2.30	−0.65	−0.95		−6.80	−5.80	−4.60	−3.88	−2.90
% value of mean dev.±	0.48	0.61	0.59	0.97	1.13		0.90	0.85	0.41	0.29	0.28

magnifying lens is placed before one of the eyes. The analytic curves defining the area merely shift their position on the vertical axis. The displacement of the region indicating fixation disparity is not so apparent as that for Mr. Ames, probably due to the difference of the actual magnitude of the fixation disparity.

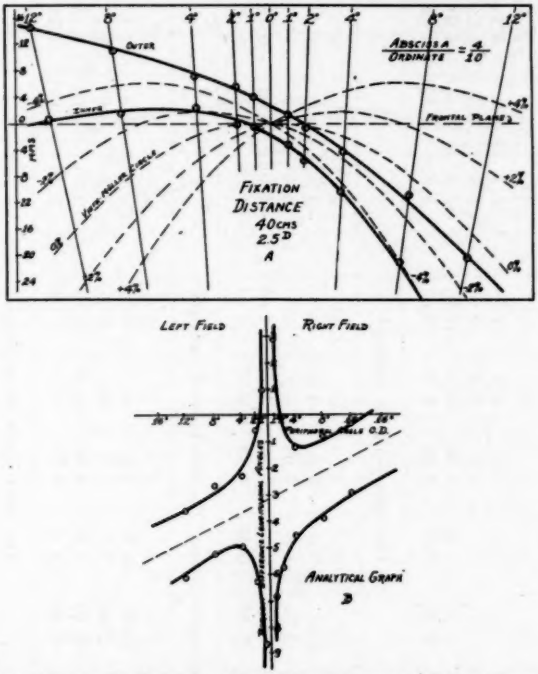


FIG. 44. *The spatial and analytic graphs of the region of binocular single vision as affected by a 3 percent magnifying lens placed before the right eye. Data of K. N. Ogle.*

The significance of the above findings is of great importance. First, many investigators have determined empirically that binocularly seen directional values are different from monocularly seen directional values. This has led to the belief that a binocular space evaluation exists which is different in nature from monocular space evaluations. The phenomenon of "fixation disparity" would indicate that the binocular space evaluation results from monocular space evaluations as effected by the disparity that may be present. If the monocular directional evaluation is determined by the spacing of points lying on the horopter as found by the grid-nonius method, the binocular directional evalua-

tions are established at the same time. In other words a full knowledge of space evaluation is obtained from monocular space evaluation if determined on the horopter as found by the grid-nonius method (see following section). Second, they show that the criterion of the frontal plane is not a reliable one for determining the position in space of the intersects of the optical projections of corresponding points. In other words the setting of an apparent frontal plane even in the case of black threads against a white background, does not necessarily give a true horopter. It may conform to the true horopter in cases where there is no "fixation disparity" but such a confirmation in no way substantiates the claim of the criterion of the frontal plane as a true one for determining the horopter.

The magnitudes of the Hering-Hillebrand deviation and the relative size of the ocular images as determined by the criterion of the apparent frontal plane, however, agree to some extent with some of those determined by the grid-nonius method (i.e., criterion of the constancy of visual direction). See Figs. 48, 49 and 50 in the following section.

The results presented in this section, especially those concerned with the difference between the true horopter and the apparent frontal plane, and with the phenomenon of retinal slip have an important significance in the general theory of binocular vision and depth localization and may lead to a revaluation of numerous concepts related thereto. These will not be discussed here, however.

C. PRELIMINARY MEASUREMENTS ON THE MONOCULAR RELATIVE DIRECTIONAL VALUES AND THE HOROPTER

The fundamental hypothesis which has been stressed in this paper and upon which the determination of the horopter rests is that any particular pair of corresponding points by innervating a single brain center gives the subjective sensation of a particular directional value. Moreover this directional value is the same whether that brain center is innervated by both of the corresponding points or by either alone. Therefore it follows that a relation must exist between the optical projections of retinal elements of each eye and directional values and that these two relations for each eye must correlate through the horopter. Experimentally this implies that the monocular partition values for each eye must be correlated through the horopter. Schematically this can be illustrated in Fig. 45 where the eyes are fixated upon F. If now points P and Q are arranged so that the angles α_2 and α_2' to the O.D. give equal subjective directional separations with respect to the fixation

point monocularly and if the horopter is known (explicitly H), then the angles α_1 and α_1' can be found and they should give the same subjective directional separation monocularly for the O.S. While only preliminary experiments have been made to test this relation and though not conducted altogether correctly they are entirely satisfactory and for that reason will be briefly described here. The haploscope was used, on the targets of which a series of seven short vertical black wires was supported so that their separations could be varied by screws.[33] The center wire was used for fixation. Monocularly these wires were adjusted until

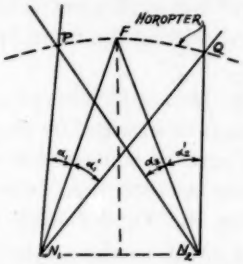


FIG. 45. *Diagram for illustrating the relation between monocular partition values and the horopter.*

TABLE 7. *Monocular multiple partition measurements for K.N. Ogle fixation distance 40 cm H grid-nonius=0.113.*

	Left field				Right field		
Wire number	3	2	1	0	1	2	3
Position of wire—O.D.mm	−37.2	−24.2	−12.0		11.3	22.7	34.0
Angles to O.D. α_2	−5.314°	−3.466°	−1.718°		1.613°	3.256°	4.853°
Percent difference in angles from horopter	−1.051	−0.684	−0.339		0.318	0.641	0.959
Corresponding angles α_1	−5.393°	−3.490°	−1.724°		1.608°	3.235°	4.807°
Corrected exp. values—O.S. mm	−38.0	−24.75	−12.0		11.45	23.0	34.8
Experimental angles α_1	−5.424°	−3.542°	−1.719°		1.639°	3.291°	4.910°

all seven appeared equally separated and the average angles from a series of settings to each eye were determined. The grid-nonius horopter was determined separately and the Hering-Hillebrand deviation value H found. These data for Dr. Ogle are shown in Table 7. With the angles found in this manner for the O.D., the corresponding angles were computed for the O.S. from the horopter[16] data. These angles, when

[33] The proper method would be to use an adjustable horopter apparatus as recently designed by Tschermak, cf. Ber. über die ges. Physiol., *61*, May, 1931.

compared with those of the monocular multiple partition values of the left eye are found to check closely. Several errors of secondary importance enter into this manner of demonstration.

It is of great importance to note that the type of partition for the right eye is that of Münsterberg while that for the left is that of Kundt. It is the greater values of the Kundt type that predominate and determine the *H* of the horopter. It should be stated in passing that only *H* as found by the grid-nonius method satisfies the calculations. This entire subject will be considered in the near future.

There is a point in connection with binocular division of a line which should be mentioned here. If the line being divided lies on the horopter the binocular division will be in conformity with the monocular space division. If, however, the line does not lie on the horopter this will no longer be true. The same objective geometrical value of position and the subjective functional value will not be found in the two types of division.[34] It would seem that the difference may be explained as follows: If the line does not lie on the horopter, points on it will be imaged on disparate retinal points in the two eyes. As has been pointed out[35] such images, if they are fused, will take the apparent direction not of the retinal elements that are directly stimulated but the direction that is associated with a near pair of corresponding points. This problem is now under study and will be quantitatively checked by various measurements of the divisions of lines both monocularly and binocularly and with the line both on and off the horopter.

D. THE NATURE OF THE CHANGE IN THE RELATIVE PROJECTIONS OF CORRESPONDING RETINAL POINTS WITH FIXATION DISTANCE

The apparatus described above together with the method of representing and examining data permits one to obtain information concerning the relative change in the projections of corresponding points with a change in fixation distance. For the procedure four fixation distances were chosen *viz.*, 20, 40, 76.5, and 609.6 cm, at each of which the trace of the longitudinal horopter was determined by the apparent frontal plane and grid-nonius methods. The results presented here are those of Dr. Ogle, Mr. Ames and Mr. Comfort. The data of Dr. Ogle are shown complete for each of the fixation distances in Table 8 and Figs. 46 and 47 give the spatial and analytical graphs for both apparent frontal plane and the grid-nonius horopter. (The grid-nonius horopter could not be determined at 6.1 meters for the reason previously given.)

[34] Reference 15, pp. 177–202.

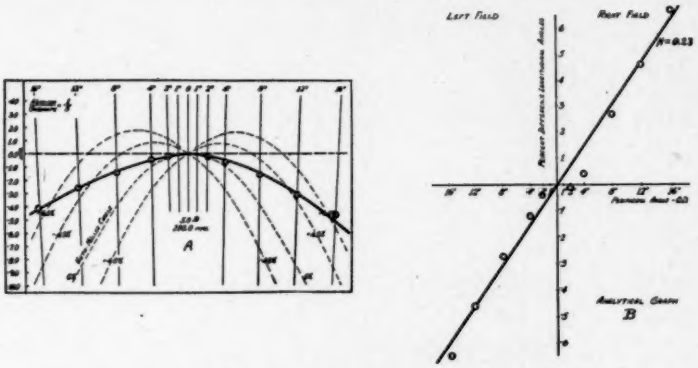
[35] J.O.S.A., *22*, pp. 552, 558; 1932.

TABLE 8. *A. The experimental data of K.N. Ogle pertaining to the change of the apparent frontal plane with fixation distance. No Rx.*

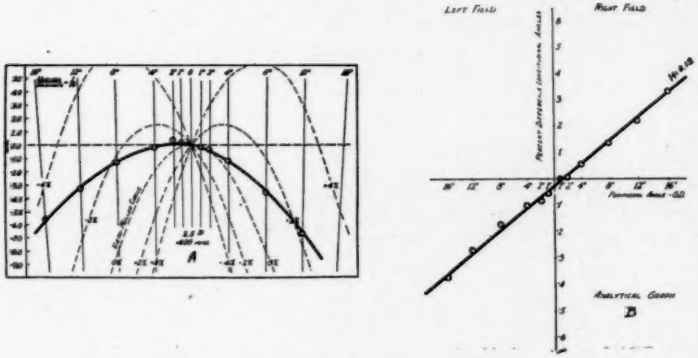
	Left field							Right field					
(a) Fixation distance of 20.2 cms (5.00 diopters) (19 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−4.07	−2.70	−1.44	−0.45	−0.17				−0.29	−0.72	−1.56	−3.04	−4.68
Mean dev.±	0.80	0.43	0.23	0.16	0.11				0.09	0.16	0.27	0.47	0.78
% value from iso-mag. curve	−6.48	−4.55	−2.73	−1.20	−0.35				−0.20	+0.40	+2.65	+4.50	+6.60
% value of mean dev.±	0.50	0.4	0.25	0.35	0.5				0.4	0.35	0.3	0.5	0.5
(b) Fixation distance of 40.4 cm (2.50 diopters) (82 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−5.45	−3.20	−1.35	−0.19	0.26	0.11		−0.14	−0.32	−1.16	−3.38	−6.56	−9.64
Mean dev.±	1.45	0.82	0.48	0.23	0.14	0.12		0.11	0.21	0.31	0.59	0.89	1.62
% value from iso-mag. curve	−3.73	−2.75	−1.84	−1.02	−0.85	−0.60		0.00	0.02	0.47	1.29	2.15	3.25
% value of mean dev.±	0.20	0.16	0.14	0.13	0.16	0.34		0.31	0.25	0.18	0.18	0.17	0.22
(c) Fixation distance of 75.6 cm (1.33 diopters) (15 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−10.95	−4.03	−2.08	−1.17	−0.98	−0.57		0.36	1.11	0.50	−0.70	−5.20	−7.20
Mean dev.±	2.3	1.6	1.5	0.7	0.4	0.3		0.3	0.5	0.6	1.6	2.7	2.5
% value from iso-mag. curve	−2.00	−1.60	−1.03	−0.40	0.04	0.22		0.36	0.64	0.68	1.16	1.54	2.20
% value of mean dev.±	0.08	0.09	0.11	0.12	0.13	0.18		0.20	0.15	0.08	0.12	0.14	0.09
(d) Fixation distance of 609.6 cm (0.164 diopters) (5 readings)													
Lane	12°	8°	6°	4°	2°	1°	0	1°	2°	4°	6°	8°	12°
Reading—cm	2.2	0.7	0.1	0.4	0.3	0.2		0.2	0.4	0.9	0.8	1.0	1.4
% value from iso-mag. curve	−0.31	−0.19	−0.12	−0.13	−0.11	−0.11		0.08	0.14	0.18	0.17	0.22	0.27
% value of mean dev.±	0.01	0.05	0.02	0.07	0.10	0.07		0.04	0.07	0.05	0.05	0.05	0.07

B. The experimental data of K.N. Ogle pertaining to the change of the grid-nonius longitudinal horopter with fixation distance.

	Left field							Right field					
(a) Fixation distance of 20.2 cm (5 diopters) (22 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm	−7.62	−4.50	−2.12	−0.36	−0.33				−0.67	−0.96	−2.10	−6.3	−10.6
Mean dev. ±	1.61	1.34	1.13	0.78	0.45				0.30	0.53	0.99	1.91	2.16
% value from iso-mag. curve	−4.45	−3.20	−1.93	−1.30	0.60				−2.00	0.00	2.05	1.85	2.75
% value of mean dev. ±	1.0	1.1	1.3	1.8	2.1				1.35	1.25	1.2	1.5	1.3
(b) Fixation distance of 40.4 cm (2.50 diopters) (15 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm		−5.5	−3.8	−2.3	−1.9	−1.7		−1.4	−2.1	−2.7	−4.1	−8.1	
Mean dev. ±		2.8	1.6	0.9	0.6	0.4		0.4	0.6	0.8	1.8	4.0	
% value from iso-mag. curve		−2.35	−1.28	0.18	1.70	3.60		−2.85	−1.83	−0.45	1.07	1.88	
% value of mean dev. ±		0.39	0.47	0.53	0.68	1.00		1.00	0.68	0.50	0.52	0.56	
(c) Fixation distance of 75.6 cm (1.33 diopters) (22 readings)													
Lane	16°	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°	16°
Reading—mm		−5.90	−3.37	−2.87	−2.57	−2.36		−0.98	−1.70	−1.60	−3.45	−8.07	
Mean dev. ±		3.39	3.85	2.75	1.73	0.89		0.94	1.07	1.90	2.94	3.56	
% value from iso-mag. curve		−1.49	−0.94	−0.12	0.55	1.4		−0.54	−0.28	0.33	0.93	1.38	
% value of mean dev. ±		0.19	0.33	0.44	0.56	0.62		0.64	0.34	0.30	0.24	0.20	



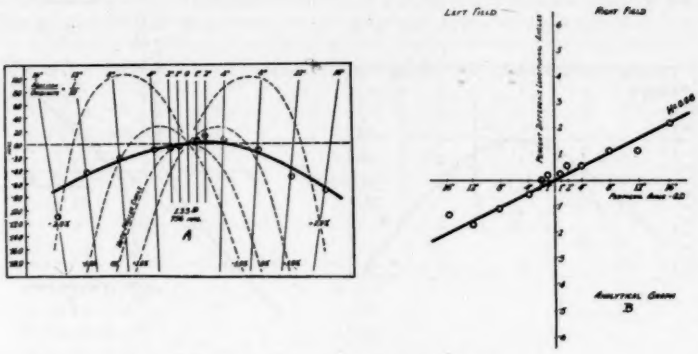
(a) *Fixation distance of 20 cm, 5 diopters.*



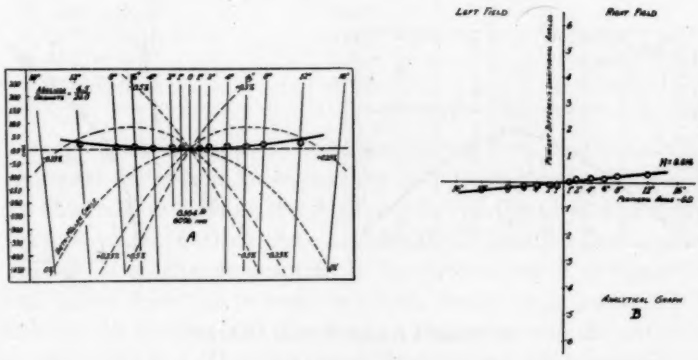
(b) *Fixation distance of 40 cm, 2.5 diopters.*

FIG. 46. *The spatial and analytic graphs of the apparent frontal plane for four fixation distances. Data of K. N. Ogle.*

A study of the graphs shows that at 6.1 meters his horopter lies very near the frontal plane. A comparison of its position with the iso-magnification curves, shows a variation from it by not more than a 0.3 percent difference of longitudinal angles. With a nearer fixation distance the departure of the horopter from the frontal plane steadily increases until with a fixation distance of 20 cm it varies from it by 2 to 3 percent. This shows conclusively that there is a change in the relative spacing of lines in the projection fans with change of fixation distance. In all cases there are the characteristic differences before described between the positions of the horopters as determined by the criterion of the frontal



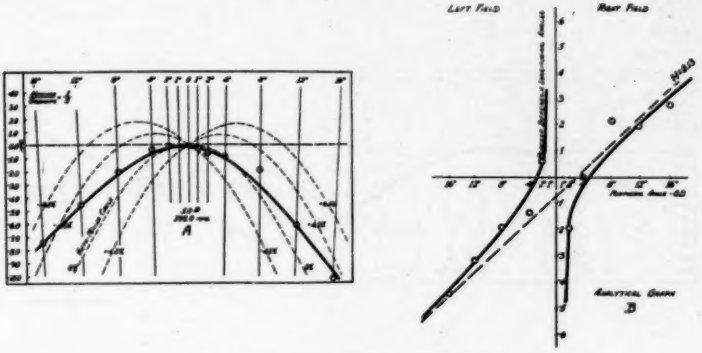
(c) *Fixation distance of 75.6 cm, 1.33 diopters.*



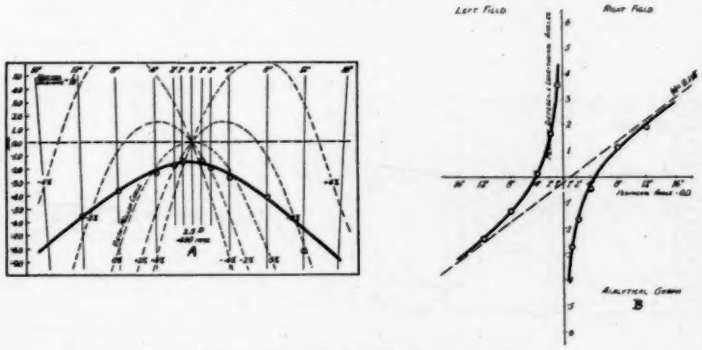
(d) *Fixation distance of 609.6 cm, 0.164 diopters.*

FIG. 46 (*Continued*)

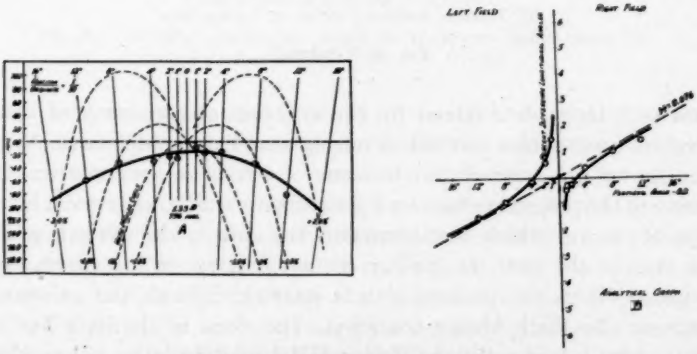
plane and those determined by the criterion of constancy of visual direction (grid-nonius method) resulting from fixation disparity. As has been shown, the quantitative measure of a relative asymmetrical distortion of the projection fans for a given fixation distance is given by the slope of the line which best represents the data in the analytic graph. The steeper the line, the greater the asymmetry. If the curves were horizontal lines the projection fans passing through the axis would represent the Vieth-Müller condition. The slope of the data line expresses what is here called the Hering-Hillebrand deviation value, which is given on the upper end of each curve, and in Table 9.



(a) *Fixation distance at 20 cm—5 D.*



(b) *Fixation distance at 40 cm—2.50 D.*



(c) *Fixation distance at 75.6 cm—1.33 D.*

FIG. 47. *The spatial and analytic graphs of the grid-nonius longitudinal horopter for three fixation distances. Data of K. N. Ogle.*

TABLE 9. *Tabulation of the experimental values of H, the quantitative measure of the Hering-Hillebrand deviation, for the apparent frontal plane and the grid-nonius longitudinal horopter.*

Fixation distance	Apparent frontal plane	Grid-nonius horopter
(a) K.N. Ogle		
(0.164 *D*) 609 cm	0.016	—
(1.33 *D*) 75.6	0.080	0.076
(2.50 *D*) 40.0	0.122	0.113
(5.00 *D*) 20.0	0.226	0.132
(b) A. Ames, Jr.		
(0.164 *D*) 609 cm	0.013	—
(1.33 *D*) 75.6	0.070	0.046
(2.50 *D*) 40.0	0.136	0.091
(5.00 *D*) 20.0	—	—
(c) F. D. Comfort		
(0.164 *D*) 609 cm	0.007	—
(1.33 *D*) 75.6	0.056	0.050
(2.50 *D*) 40.0	0.110	0.097
(5.00 *D*) 20.0	0.186	0.136

A complete and comprehensive understanding of the manner in which the horopter varies from far to near can be obtained from a study of the curve obtained by plotting the slopes of the curves at which they were found against the fixation distance (diopters). Such a curve is shown in Fig. 48. If no change occurred in the projections of corresponding retinal points from far to near the curve would be a horizontal line parallel to the abscissa axis. Insofar as a change occurs, the curves deviate from that line. The light dashed line on the graph shows what the change in the relative asymmetry of the projection fans would have to be if the horopter coincided with the fixation frontal plane at all distances. The empirical data presented show that the variations in the horopter with change in distance are such as to cause it to be nearer the frontal plane at all distances than it would be if there were no variations. The point of intersection of the data curve and the dashed line indicates that distance at which the horopter coincides with the frontal plane.

The Hering-Hillebrand deviation values for Mr. Comfort and Mr. Ames for the different fixation distances and for the apparent frontal plane and grid-nonius are also shown in Table 9, and Figs. 49 and 50 show the data graphically. Their similarity with that for Dr. Ogle is evident.

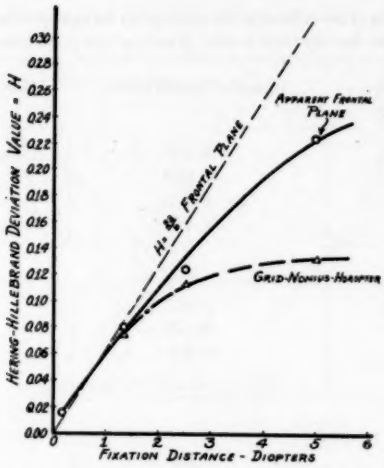


FIG. 48. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane and the grid-nonius horopter with a change in the fixation distance. Data of K. N. Ogle.*

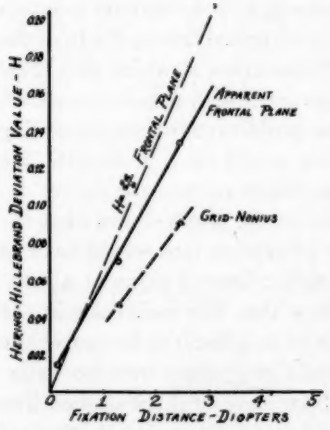

HERING-HILLEBRAND DEVIATION VALUE - H
H = 1/2 FRONTAL PLANE
APPARENT FRONTAL PLANE
GRID-NONIUS
0 1 2 3 4 5
FIXATION DISTANCE - DIOPTERS

FIG. 49.

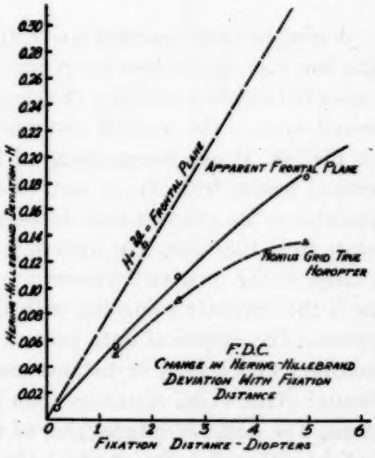


FIG. 50.

FIG. 49. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane and the grid-nonius horopter with a change in the fixation distance. Data of A. Ames, Jr.*

FIG. 50. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane and the grid-nonius horopter with a change in the fixation distance. Data of F. D. Comfort.*

It should also be noted that in all three cases the change in the relative asymmetry of the projection fans, given by the Hering-Hillebrand horopter deviation value is different for the two methods used. That found by the criterion of the apparent frontal plane is nearer the true frontal plane than that found by the grid-nonius method. The only explanation that can be suggested is that the subjective interpretation of the disparity that constitutes an apparent frontal plane changes with the fixation distance. It is interesting to note that although the actual amplitude of accommodation for Mr. Ames, due to his presbyopia is much less than that of Mr. Comfort and Dr. Ogle, the characteristics of his curves are much the same.

The method of examining and studying experimental data used above is readily applicable to the results found in the literature of other observers. These data pertaining to the apparent frontal plane (horopter) at various fixation distances will be found in the work of Helmholtz,[36] Liebermann,[37] and Lau,[24] and will be considered here.

(1) Helmholtz determined the apparent frontal plane at various fixation distances, by finding that distance at which three plumbs, the center of which was displaced behind the two others, appeared to lie in a frontal plane. The data given for the separations of the three points and the fixation distance are sufficient for the case of a symmetrical horopter to allow the Hering-Hillebrand deviation value to be com-

TABLE 10. *Data for apparent frontal plane of Helmholtz and the computed value of H for each fixation distance.*

$2a$ = base line = 68 mm

Fixation distance b mm	450	730	237	129	550
" " diopters	2.22 *D*	1.37 *D*	4.22 *D*	7.75 *D*	1.82 *D*
Displacement of central thread $b-y$	10.5 mm	6.0	4.2	8.1	2.0
Separation of outer threads $2x$	256 mm	256	117	117	120
Hering-Hillebrand deviation value H	0.108	0.068	0.203	0.366	0.086

puted. His results are reproduced in Table 10,[16] and are shown graphically in Fig. 51, where H, the deviation value, and the fixation distance in diopters are plotted on the ordinates and abscissa, respectively. The linear relation between the data is striking and is wholly in accord with the data from this laboratory as determined by the criterion of the apparent frontal plane.

[36] Helmholtz, Physio. Optics, Eng. Trans., 3rd Ed. Vol. III, p. 319.

[37] Liebermann, Zeits. f. Psych. und Physiol., *44*, p. 436; 1910.

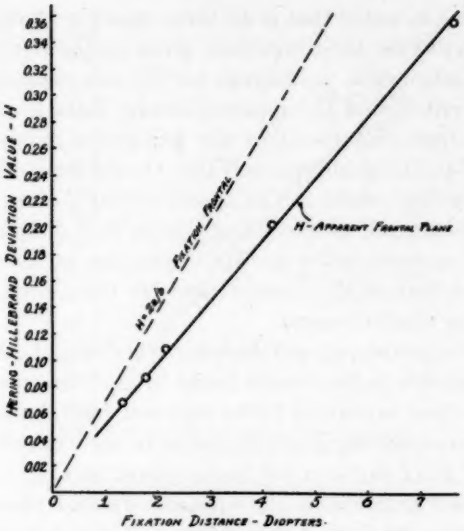


FIG. 51. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane. Data of Helmholtz.*

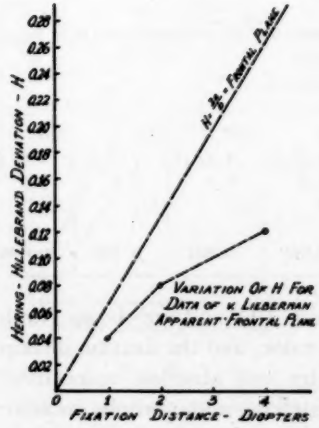

HERING-HILLEBRAND DEVIATION - H
FIXATION DISTANCE - DIOPTERS
VARIATION OF H FOR DATA OF V. LIEBERMAN APPARENT FRONTAL PLANE

FIG. 52. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane. Data of Liebermann.*

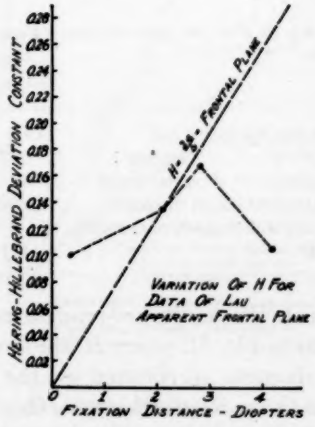

HERING-HILLEBRAND DEVIATION CONSTANT
FIXATION DISTANCE - DIOPTERS
VARIATION OF H FOR DATA OF LAU APPARENT FRONTAL PLANE

FIG. 53. *The graphical representation of the change in the Hering-Hillebrand deviation values for the apparent frontal plane. Data of Lau.*

(2) At three fixation distances Liebermann determined the apparent frontal plane (horopter) using three vertical plumbs, the center one being used for fixation. From his data complete analytical graphs were determined for each fixation distance, and from each of these the Hering-Hillebrand deviation value was found. The results are shown in Table 11 and Fig. 52.

TABLE 11. *Data for apparent frontal plane of Liebermann and the computed value of H for each fixation distance.*
$2a$ = base line = 66 mm

Fixation distance mm	250	500	1000
" " diopters	4.0 D	2.0 D	1.0 D
Hering-Hillebrand deviation value H	0.122	0.080	0.040

(3) Lau's researches on the constancy of identical points with vision at different distances was conducted in the same manner for one peripheral angle as was Liebermann's, i.e., using three plumb lines, the central one being fixated. Four fixation distances were used. His results are given in Table 12 and shown graphically in Fig. 53. Lau's results show more nearly a constancy in the relative projections of corresponding points, and are the only ones where the curve crosses the fixation frontal plane line.

TABLE 12. *Data for apparent frontal plane of Lau and the computed value of H for each fixation distance.*
$2a$ = base line = 64.5 mm

Fixation distance b mm	240	360	480	1440
" " diopters	4.17 D	2.78 D	2.08 D	0.69 D
Lateral separation x mm	6.0	9.0	12.0	36.0
Displacement of center thread from plane of outer two $(b-y)$ mm	0.09	0.015	0.00	−1.10
Mean variation	0.05 mm	0.03	0.01	0.5
Hering-Hillebrand deviation value H	0.105	0.167	0.134	0.100

The data of Helmholtz and Liebermann definitely agree with those made in this laboratory. Lau's findings do so to a lesser extent, but the irregularity in them prevents definite conclusions to be drawn one way or the other.

In view of the definiteness of these findings obtained with an apparatus and method that far excels any that has been used before, it is believed that there can be no question but that the projection fans do change with a change of fixation distance.

Changes in anatomical distribution of the rods and cones in view of the flexibility of the bulbus would not be unexpected. For instance, Hensen and Völkers[38] have shown that the choroid moves 0.5 mm at the equator of the eye due to the tension of the ciliary muscle in an effort of accommodation. The equator is at a peripheral visual angle of about 68° from the fovea. A movement of 0.5 mm at an angle of 68° would produce a change of about 3 percent in the angle of projection of a retinal point. A similar but smaller retinal movement and change of the projected angle may also take place as one approaches the posterior part of the eye.

The type of zig-zag horopter variations found by Jaensch in eidetics may be associated with variations in vergence together with an anomalous ocular image size condition. Dr. Ogle frequently experienced this zig-zag appearance in experimenting with vertical lines, when no suitable fixation object was in the field of view. It is of interest to note here in reference to Jaensch's work, that apparently an unduly large number of the patients examined in this laboratory who had a size difference were of a marked visual imagery type.

The above described determinations of the horopter at different distances brought out one other important point which should be noted. The mean variation of the settings decreased with an increase in the fixation distance. These variations, which are given in the different tables, are compiled in Table 13. In all cases there is a consistent decrease with an increase in the fixation distance. The average decrease, from 20 cm to 6.1 m, is about eightfold. The error for nearer distances may be somewhat too large, due to a decrease in mechanical leverage. This increased visual sensitivity is related to the change in visual acuity with accommodation. This has been demonstrated by recent experiments.[39]

To what extent the factors of accommodation and convergence of the eyes are related to this change in the effective distribution of corresponding elements with the fixation distance, is not yet entirely clear. Evidence would indicate that the change is correlated both with the effort of accommodation and with change in convergence. Data of the apparent frontal plane obtained with both plus and minus lenses and distortionless prisms, base-in and base-out, show different Hering-Hillebrand values. The order of this change, both with relaxation of the

[38] Hensen and Völkers, *Über den Mechanismus der Accommodation*, p. 27. Kiel, 1868.

[39] K. N. Ogle, *The Resolving Power of the Eye*, unpublished thesis, 1930. See also: Kirsch, f.O.G., *103*, p. 253; 1920; and Biedermann, Z. f. o. Opt., *15–16*, pp. 1 and 34; 1927–28.

TABLE 13. *Tabulations of the mean deviations from mean settings of the wires for the apparent frontal plane and the grid-nonius longitudinal horopter at the various peripheral angles.*

Fixation distance diopters	Mean deviation for peripheral angle Seconds of arc						Number of readings
(a) K.N. Ogle. Apparent frontal plane							
	1°	2°	4°	8°	12°	16°	
0.164 (6 m)	—	5″	5″	11″	13″	—	18
1.33 (75.6 cm)	7″	10″	14″	35″	52″	67″	70
2.50 (40 cm)	12″	15″	22″	46″	71″	120″	240
4.94 (20.2 cm)	—	32″	62″	89″	170″	320″	82
(b) F.D. Comfort. Apparent frontal plane							
	1°	2°	4°	8°	12°	16°	
0.164 (6 m)	3.4″	5″	9″	20″	28″	—	60
1.33 (75.6 cm)	14″	22″	32″	61″	110″	190″	10
2.50 (40 cm)	20″	39″	72″	170″	310″	—	104
4.94 (20.2 cm)	—	115″	140″	220″	450″	590″	12
(c) K.N. Ogle. Grid-nonius longitudinal horopter data							
	1°	2°	4°	8°	12°	16°	
1.33 75.6 cm	20″	35″	50″	90″	110″	—	22
2.50 40 cm	30″	40″	70″	138″	280″	—	450
4.94 20.2 cm	—	145″	290″	360″	570″	670″	22

accommodation and divergence is towards a smaller *H*. This technique, together with the apparatus for accurately finding the accommodative state, provides the possibility of learning more concerning (see also section VI, F) the relation of accommodation and convergence and the phenomenon of phoria.

Further evidence that the projection fans do not remain constant may be found in the statistics of clinical cases where a change in the relative size and shape of the ocular images is found in: first, difference

in relative size, near and far; second, capacity to change relative size, i.e., so-called size amplitude; third, change in relative size after corrections were worn.

Such changes in the distribution of the projection of corresponding retinal points might result either from change in the optical image formed at the retina or in the anatomical distribution of cones and rods. It would appear not unnatural that such changes might take place. Asymmetrical changes in the optical image would result from changes in curvature of the crystalline lens due to angles alpha and beta, or from a slight change in the angle beta or in the centering of the lens, relative to the cornea.

E. THE NATURE OF THE CHANGE IN THE PROJECTIONS OF CORRESPONDING RETINAL POINTS WITH ASYMMETRICAL CONVERGENCE OF THE EYES

In general the horopter is determined with symmetrical convergence of the eyes, i.e., the fixation point lies in the median plane. The change in the horopter with asymmetrical convergence, i.e., with the fixation point at one side of the median plane, was studied by Herzau,[40] who used the criterion of the apparent frontal parallel plane. His results indicated that the horopter varied from those similarly determined with symmetrical convergence and he suspected that the variations were due to an "indirect kinesthetic influence of the optic muscles." Similar measurements have been made in this laboratory. The difficulty experienced by Herzau, *viz.*, that of judging what constitutes an apparent frontal plane with asymmetrical convergence was also encountered in this laboratory. This difficulty has been avoided by the use of the grid-nonius method (the criterion of constancy of visual direction). The longitudinal horopters for Dr. Ogle, Mr. Comfort, and Mr. Ames were determined by this method while looking to the right and to the left 8° from the median plane with a fixation distance of 40 cm. The data for Dr. Ogle, and Mr. Comfort are shown in Tables 14 and 15, and Figs. 54 and 55. By comparing these data with those obtained with symmetrical convergence it is seen that the traces of the horopter determined with asymmetrical convergence lie nearer the Vieth-Müller circle, thus, the Hering-Hillebrand deviation value has decreased. The data for Mr. Ames indicated that although in the same direction the change was not as great as that obtained by Dr. Ogle and Mr. Comfort. This difference must be due to an actual change in the asymmetry of the projection fans accompanying the lateral change in fixation. The

[40] W. Herzau, Arch. f. O. G., *121*, p. 756; 1928–29.

TABLE 14. *The data for the grid nonius longitudinal horopter for K.N. Ogle with asymmetrical convergence. Fixation distance 40 cm.*

	Left field						Right field				
(a) Looking right 8° (6 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−4.0	−4.05	−1.3	−0.1	−0.4		−1.4	−2.6	−3.47	−17.2	−29.8
Mean dev. ±	4.0	3.6	2.7	1.5	0.67		0.45	0.6	1.01	2.6	3.37
% value from iso-mag. curve	−0.45	1.10	1.85	1.75	3.05		−0.65	−0.20	1.15	−0.46	−0.22
% value of mean dev. ±	0.80	1.08	1.62	1.95	1.81		0.90	0.60	0.61	0.78	0.67
(b) Looking left 8° (6 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−31.8	−20.1	−8.9	−3.83	−2.1		0.18	0.5	2.65	−0.6	−4.3
Mean dev. ±	4.68	2.57	3.13	1.07	0.72		0.45	0.92	1.30	1.8	1.52
% value from iso-mag. curve	0.65	1.35	1.65	1.52	2.00		−1.5	−1.05	0.35	−0.10	−4.0
% value of mean dev. ±	0.94	0.77	1.88	1.07	1.44		1.22	1.20	0.78	0.54	0.30

TABLE 15. *Data for the grid-nonius longitudinal horopter for F.D. Comfort for asymmetrical convergence. Fixation distance 40 cm.*

	Left field						Right field				
(a) Looking right 8° (5 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−23.02	−13.5	−0.38	3.34	2.36		5.36	6.92	9.72	10.78	10.06
Mean dev.±	5.10	2.52	1.30	0.91	1.63		1.27	2.26	2.22	1.38	4.81
% value from iso-mag. curve	−1.20	−0.60	−3.12	−6.45	−9.60		8.95	5.60	4.10	3.15	2.75
% value of mean dev.±	1.02	0.76	0.78	1.18	4.41		2.54	2.26	1.33	0.42	0.96
(b) Looking left 8° (5 readings)											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	0.98	3.82	7.18	5.38	4.54		1.96	1.18	−2.46	−10.14	−18.06
Mean dev.±	3.38	3.86	1.78	0.90	1.31		1.01	1.61	1.71	1.66	4.57
% value from iso-mag. curve	−1.40	−1.10	−2.80	−4.20	−7.50		8.25	4.20	1.92	1.57	2.20
% value of mean dev.±	0.68	1.16	1.07	0.90	2.62		2.72	2.10	1.03	0.50	0.91

curves in Fig. 54 show first, the position of the points in the longitudinal horopter determined with asymmetrical convergence of 8°, and second, the loci of the intersects of the same projection fans that were found

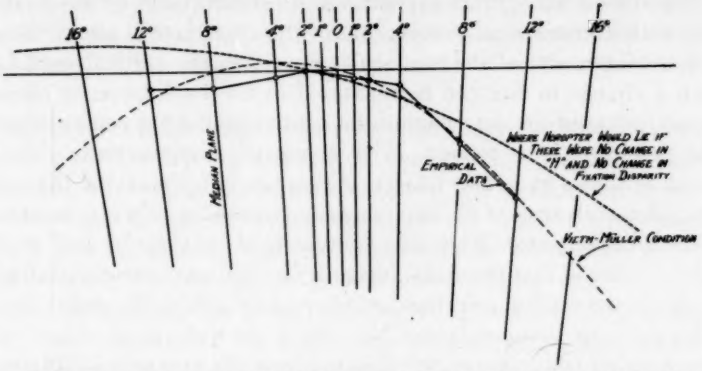


FIG. 54. *The spatial representation of the data of K. N. Ogle for the grid-nonius longitudinal horopter for an asymmetrical convergence of 8° to the right.*

with symmetrical convergence, i.e., using the same Hering-Hillebrand deviation value. It will be seen that the position of the latter curve is quite different from the positions found empirically. The close agree-

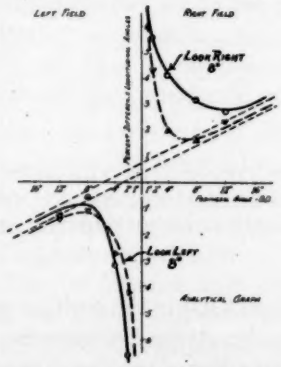


FIG. 55. *The analytic graph of the data of F. D. Comfort for the grid-nonius longitudinal horopter for asymmetrical convergence of 8° to the right and to the left.*

ment of the findings of the three observers indicates their reliability. Since the only criterion used was that of equating directional values, Herzau's explanation would not seem to be applicable. The explanation would seem to be that the optical projection of such points changed

either through variations in the characteristics of the dioptric system or through an anatomical movement of corresponding retinal points or through both.

That this is the correct explanation is substantiated by the finding that with asymmetrical convergence of the eyes there is also a change in the relative size of the two ocular images in the vertical meridian. Such a change in size can be measured in the haploscope or clinical apparatus[1] used for determining the relative size of the ocular images. The procedure is as follows. With symmetrical convergence the observer equalizes his ocular images, if they are not already of the same size. Then both arms of the haploscope are rotated to give him an asymmetrical convergence. When this asymmetrical convergence is 8° to the right, it is found that the ocular image of his right eye becomes relatively smaller in the vertical meridian and when to the left 8°, the ocular image of his right eye becomes larger. Now when one looks at an object with asymmetrical convergence, the distance from the two eyes is different. The image of such an object formed in the nearer eye will be larger than that formed in the other. Now it happens that with the particular set-up chosen for the measurements described above, the difference (about 2 percent) found in the relative sizes of the ocular images in the vertical meridian is almost exactly the amount that would be necessary to make up for the difference in size of the two images due to the difference in distance. This may be only a coincidence. A careful check was made to see if the effect could be accounted for in any other way, without success. It is a very apparent phenomenon and has been found by a number of observers.

F. THE NATURE OF THE CHANGES OF THE EFFECTIVE PROJECTION OF CORRESPONDING RETINAL POINTS AND THEREFORE THE HOROPTER PRODUCED BY OPHTHALMIC LENSES

Introduction

With any given fixation distance the position of the loci of the intersects of the optical projections of the corresponding points of the two eyes is determined by the distribution of the lines in the projection fans of the two eyes. Anything that affects the distribution of these projection lines will alter the position of their intersects and therefore of the horopter. Any lens in the paths of these projections which, through its refractive effect deflects rays which pass through it, will alter the position of such projections. That is, it will alter the *effective* projection of corresponding retinal points and therefore the horopter.

This section of the paper will describe typical changes in the effective shape and position of the projection fans and horopter produced by the use of ophthalmic lenses. The only investigation of the influence of such lenses upon the horopter and space perception that has been found in the literature is that of F. P. Fischer, which is reported by Hartinger[41] and Fischer.[42] The results obtained apparently were not clearly understood.

The nature of the change in the projection lines resulting from putting a lens in their path is governed by the optical properties of the particular lens used. If these properties are known, the change that they will produce can be exactly calculated. Such calculated changes should check with the changes that are experimentally determined with the lens before the eye. In general this was found to be the case.

In order to make the study quantitative, it was necessary to determine the properties of the lenses used with respect to their effect in deflecting rays which pass through them. Such properties of lenses can be calculated. In view of the doubt as to whether such calculated values would give the actual effective values when such lenses were placed before the eyes, their distortion characteristics were also determined by measurement. For this purpose a special lens-measuring apparatus embodying some of the features of a lens bench was designed.[43] Briefly, it may be described as follows. An adjustable holder for the lens is mounted at a distance from a central point of the apparatus about which the holder can rotate. This central point corresponds to the mean nodal point of an average eye. The distance from the lens holder to this point is the same as that between the anterior focal point and the mean nodal point of the eye. A telescope is also mounted in such a way that it too can move about this mean nodal point through angles up to 30°, with its optical axis passing through the point. Upon a track in alignment with the central point opposite the telescope, a point source of light is set up on a slide, so that its distance from the central point can be varied. With no lens in the holder, and the telescope set for zero position, the point light source and the central point lie upon the axis of the telescope, as determined by suitable cross hairs in the latter. A diaphragm corresponding to the entrance pupil of the eye is mounted between the lens and the central point at approximately the correct posi-

[41] H. Hartinger, Central-Ztg. f. Optik u. Mechanik, 21–27; 1923.

[42] F. P. Fischer, Arch. f. O. G. *114* (3/4), pp. 441–464; 1924.

[43] For complete description see *Apparatus for Measuring Effective Distortion of Lenses Placed before the Eyes*, A. Ames, Jr., G. H. Gliddon, and Kenneth N. Ogle. To be published.

tion. Now with a lens in the holder and the point light source at a given distance, any change in direction of the rays produced by the lens can be measured by the rotation of the telescope necessary to bring the image of the light point back to the cross hairs. If the lens in the holder is rotated to a given angle from the zero axis of the apparatus, the change in the direction of the rays coming through the lens at that angle is easily measured. Eliminating the prismatic effect, these changes of direction are recorded as percent changes for the given oblique ray. The apparatus has been so constructed that the effective distortion of lenses

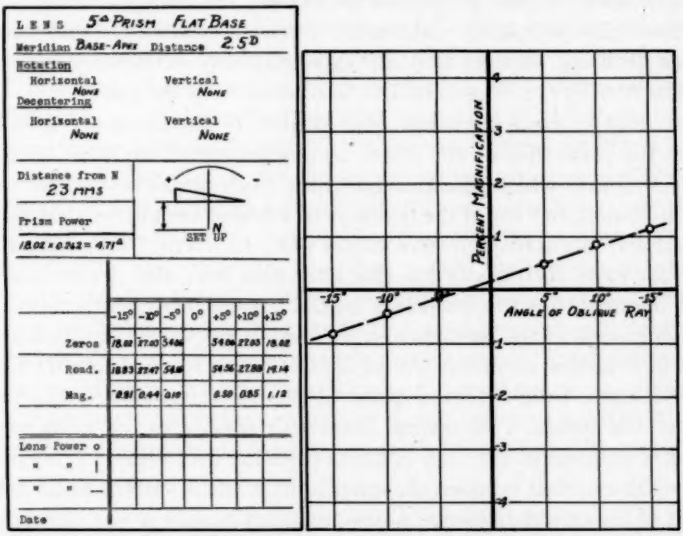


FIG. 56. *A typical example of the data and distortion curve of a lens. In this case they represent the data of a 5 diopter prism on a flat base.*

can be determined to within an error of 0.02 percent of the oblique angle, the lens power within an error of 0.02D and the prism power within 0.10 Δ. The method of graphing the distortion curve of a lens is as follows. The angular obliquity of a ray is plotted on the abscissa against the percentage change in angle produced by the lens on the ordinate. The percentage increase in angle is plotted above the abscissa axis. Rays entering the lens from the right field are plotted to the right of the ordinate axis, those from the left to the left. One can speak of the distortion curves[16] when they are for various meridians of a given lens. An example of the distortion curve of lenses is given in Fig. 56, which

shows the typical distortion produced by a flat prism. There is the typical increase in distortion towards the apex of the prism. A slight magnification effect is produced at the axis of the lens as is shown by the curve intersecting the ordinate above the abscissa-axis. The nature of these graphs is the same as that of the analytical horopter graphs shown in the preceding sections. The latter graphs showed the percentage difference in the angles formed by corresponding lines of the projection fans of each eye. These graphs show the difference in the angles of the projection lines of the fan of a theoretical eye, before and after a lens has been placed in front of it. This similarity of nature of these two types of graphs makes it possible to obtain a graph of the theoretical effect that a lens should have on the horopter, by simply adding the curve giving the distortion of a lens to the curve of the determined horopter on the analytical graph.

In general, the method of procedure in determining the effect of lenses on the horopter is as follows. The longitudinal horopter of a given observer is first determined without glasses and the data put in graph form by one of the two methods described. A lens, whose effect on the horopter is to be determined, is then carefully positioned before one eye and the horopter redetermined and plotted as before. The difference in the position of the two horopter curves should be equal to the measured distortion effect of the lens. As stated, this was found to be true in general, save where the accommodation-convergence relationship is altered.

The experimental results showing the effect of the lenses on the horopter will be grouped according to the distortion characteristics of various types of lenses and presented in the following order:

(1) Typical changes in the shape and position of the horopter produced by the use of size lenses.

(2) Typical changes in the shape and position of the horopter produced by the use of prisms.

(3) Typical changes in the shape and position of the horopter produced by the use of power lenses.

(1) Typical changes in the shape and position of the horopter produced by the use of size lenses

The ordinary overall magnification effects associated with power lenses are generally understood and will be discussed under heading (3). In so-called "size lenses" this power effect is eliminated. Lenses[1] designed by this laboratory, and made by the American Optical Com-

pany, have the effect of increasing the image overall by percentages up to about 6 percent, with little or no power. Such a lens, when measured upon the apparatus, shows approximately the same percent increase in image size for all oblique angles. Fig. 57 shows typical distortion curves of size lenses as determined by the lens measuring apparatus. As stated, the abscissae express the obliquity of the rays measured in degrees, the ordinates the distortion of the oblique rays in terms of percent of its angular distance from the axial ray.

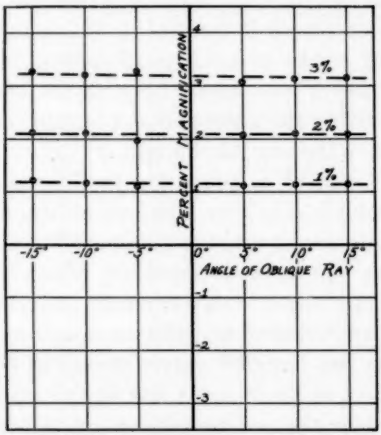


FIG. 57. *A typical set of distortion curves of size lenses.*

A study of the changes in the longitudinal horopter produced by these lenses is of importance because of their clinical value in correcting size defects without altering the refractive power of the eye. If an observer places a size lens before one eye, and by adjusting the wires of a horopter apparatus determines the longitudinal horopter by any criterion he will find that the wires will take a position in the visual field asymmetrical to the apparatus, as if rotated about the point of fixation. If the magnifying size lens is placed before the right eye, the wires in the right field will be adjusted nearer the observer and those of the left field farther from the observer. The whole visual field of single binocular vision is rotated about the point of fixation and hence gives a false basis for the interpretation of binocular depth localization. Hillebrand[44] (also Fischer,[45]) recognized that when a very thick plane parallel was

[44] Hillebrand, *Die Stabilität der Raumwerte auf der Netzhaut*, Zeits. f. Psychol. u. Physiol. d Sinnesorg. *5*, p. 47; 1893.

[45] Fischer, reference 42, p. 451.

placed before one eye, the threads of the horopter apparatus, when adjusted for the apparent frontal plane, were rotated about the central thread. He recognized, moreover, that increasing the size of the image in one eye symmetrically overall produced this rotation of the field. Erggelet[46] found in experiments with artificial anisometropia produced when a contact glass was used before one eye, that needles had to be placed in a very steep curve through the point of fixation in order that they should appear in a line in an apparent frontal plane. He apparently failed to completely understand the cause of this phenomenon, however. Brown and Kronfeld[47] recognized this rotation of the horopter

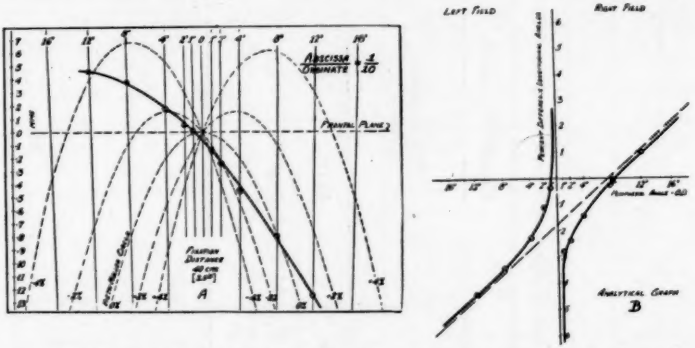


FIG. 58. *The spatial and analytic graphs of the grid-nonius longitudinal horopter obtained when a 1.5 percent magnifying lens (zero power) is placed before the right eye. Data of K. N. Ogle.*

as due to an asymmetry of images. Thus the determination of the position of the longitudinal horopter becomes a sensitive means for detecting the existence of any inequality in overall size of the ocular images in the longitudinal meridian of the two eyes. It has been shown that the amount of rotation of the horopter has a direct and calculable relation to the difference in size of the images of the two eyes. Below are presented three sets of experimental data where the ocular image in one eye has been increased by the use of the size lenses. Both the spatial and analytically transformed data are shown graphically.

Table 16 gives the calculated and experimental values of the apparent rotation of the visual field when a difference in the overall size of one ocular image is produced by size lenses with different fixation distances. Table 17, Fig. 58A shows the rotation of Dr. Ogle's horopter, produced

[46] H. Erggelet, Kl. M.f.A. *66*, p. 688; 1921.

[47] E. V. L. Brown, and P. C. Kronfeld, Arch. of Ophthalm. *5*, p. 635; 1930.

TABLE 16. *Angular rotation of field in degrees with fixation distances indicated.*

Difference in size of dioptric images of the two eyes	Theoretical values for 65 mm P.D.			Experimental checks								
				K.N. Ogle 63 mm P.D.			F.D. Comfort 63 mm P.D.			A. Ames, Jr. 68 mm P.D.		
	40 cm	75.6 cm	6 m	40 cm	76.5 cm	6 m	40 cm	76.5 cm	6 m	40 cm	76.5 cm	6 m
0.5%	1.8°	3.3°	25.1°	—	—	—	—	—	22.4°	—	—	—
1.0%	3.5°	6.6°	43.1°	4.2°±1.2°	7.2°±0.9°	40.0°	2.7°±1.0°	5.8°±1.4°	—	4.1°	6.5°±0.3°	32.2°
2.0%	7.0°	13.0°	61.7°	7.2°	—	—	5.5°	—	—	—	12.4°±0.6°	—
3.0%	10.4°	19.0°	70.2°	9.5°±1.7°	15.6°±2.3°	—	10.2°±1.1°	16.3°±1.4°	—	—	17.8°±0.2°	—

TABLE 17. *The spatial and analytic graphs for the grid-nonius longitudinal horopter when a 1.50 percent magnifying lens (zero power) is placed before the right eye. Data of K.N. Ogle. (13 Readings.)*

	Left field						Right field				
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	4.4	3.8	1.9	0.5	0.0		−1.4	−2.5	−4.5	−8.0	−12.6
Mean dev.±	1.76	1.32	0.79	0.60	0.42		0.29	0.36	0.64	1.88	2.76
% value from iso-mag. curve	−4.3	−3.4	−2.2	−1.1	−0.4		−2.9	−2.4	−1.5	−0.0	1.0
% value of mean dev.±	0.33	0.38	0.47	0.73	0.95		0.66	0.44	0.38	0.54	0.52

TABLE 18. *The spatial and analytic graphs for the apparent frontal planes when 5 diopter prisms flat base are placed before the eyes base in and base out. Data of A. Ames, Jr. (5 Readings.)*

	Left field						Right field				
(a) Base in											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	−17.8	−9.06	−2.56	−1.10	−0.32		−0.12	0.24	0.30	−3.60	−8.64
Mean dev.±	1.4	0.9	0.6	0.5	0.2		0.15	0.3	0.7	0.9	1.7
% value from iso-mag. curve	0.10	0.37	0.35	0.70	0.40		0.60	0.85	1.30	1.22	1.75
% value of mean dev.±	0.25	0.25	0.32	0.55	0.40		0.30	0.30	0.40	0.25	0.32
(b) Base out											
Lane	12°	8°	4°	2°	1°	0	1°	2°	4°	8°	12°
Reading—mm	4.48	1.04	1.04	0.24	0.40		0.06	−0.10	0.40	0.06	2.52
Mean dev.±	1.5	1.0	0.4	0.2	0.1		0.1	0.3	0.6	1.3	2.5
% value from iso-mag. curve	−4.10	−2.50	−1.71	−0.85	−1.25		0.45	0.45	1.37	2.29	3.93
% value of mean dev.±	0.28	0.30	0.20	0.20	0.30		0.30	0.30	0.32	0.40	0.46

by a 1.50 percent magnifying lens placed before his O.D. For instance, Fig. 37 shows the analytical graph of Dr. Ogle's normal horopter, 40 cm fixation distance; Fig. 58B shows the analytical graph with a 1.50 percent size lens in front of the right eye. The difference between the places where these two curves cut the ordinate axis, which specifies the difference in the size of the ocular images produced by the lens, is about 1 50 percent. See also Fig. 44, showing the rotation of the region of binocular single vision.

The increase in the magnitude of the rotation for more distant fixation distances for the same percent difference in size, is of great importance.[10] It should be emphasized at this point, that the analytical transformation of the experimental data permits one to determine at once the quantitative magnitude of any inequality in the overall longitudinal size of the ocular images that may exist.[48] The magnitude of any vertical size difference between the two eyes could be found in the same way with the grid-nonius method. At present it is only found at one peripheral angle by the haploscopic method described elsewhere.

When, due to a difference in the size of the ocular images the horopter rotates about the point of fixation, the region of binocular single vision, as limited by the size of Panum's fusional areas, also rotates about the point of fixation.[10] As the percent difference in size is increased, the magnitude of the rotation increases so that objects in space which lie near the frontal plane, may lie outside the region of single vision for larger peripheral angles. As the size difference increases the peripheral angle at which doubling occurs will decrease eventually reaching a minimum value, *viz.*, the width of the region at the fixation point. Here doubling does not occur, even with very large size differences. This large foveal region where doubling does not occur was experienced by Erggelet.[46] He found fusion only at the fovea with a high degree of anisometropia produced with a contact glass on one eye, which would cause a larger difference in the size of the ocular images.

(2) Typical changes in the shape and position of the horopter produced by the use of prisms

A consideration of the distortion of prisms is a study in itself, and will be discussed only briefly here. In general, the distortion produced by a

[48] Evidence was frequently found, that the apparent frontal plane failed to rotate to the full amount as demanded by theory. Presuming all empirical factors to be eliminated, the effect must be caused by an ocular image change. The effect was greatest when fixation objects which held vertical fusion were used, together with size spheres, but not with size cylinders.

prism can be said to be linear, i.e., a constant increase in image size in the direction of the apex of the prism. The magnitude of this distortion and size effect can be said to depend upon six factors:

(1) The prism (displacement) power.
(2) The base curve upon which the prism is ground.
(3) The distance of the prism from the eye.
(4) The degree of tipping with respect to the visual axis.
(5) The centering of the prism with respect to the visual axis.
(6) The lens power in the prism.

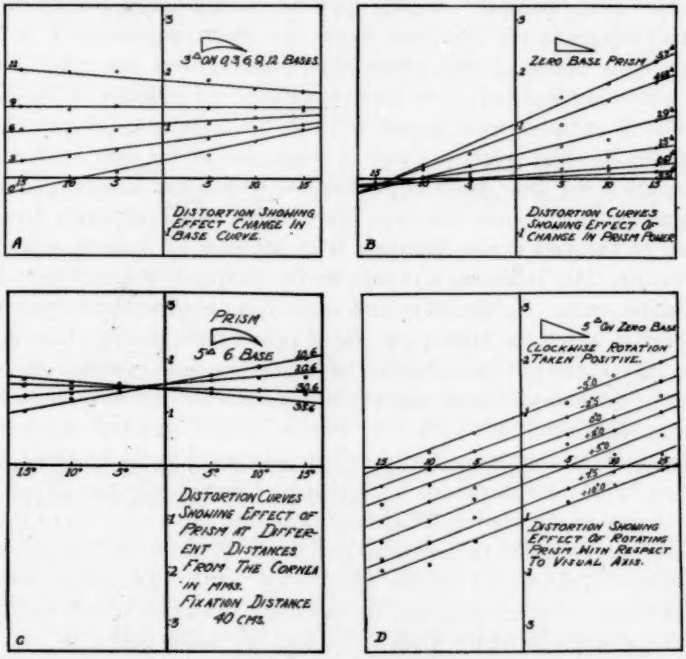

3° ON 0 3 6 9 12 BASES.
DISTORTION SHOWING EFFECT CHANGE IN BASE CURVE.
A
ZERO BASE PRISM.
DISTORTION CURVES SHOWING EFFECT OF CHANGE IN PRISM POWER.
B
PRISM
5Δ 6 BASE
DISTORTION CURVES SHOWING EFFECT OF PRISM AT DIFFERENT DISTANCES FROM THE CORNEA IN MMS.
FIXATION DISTANCE 40 CMS.
C
5Δ ON ZERO BASE
CLOCKWISE ROTATION TAKEN POSITIVE.
DISTORTION SHOWING EFFECT OF ROTATING PRISM WITH RESPECT TO VISUAL AXIS.
D

FIG. 59. *Typical distortion curves of prisms.*

For the present the last two factors will be omitted, i.e., the effect of decentered prisms and prisms with lens power will not be considered. In general, an overall size effect, depending on thickness, base curvature, and position before the eye is also produced by prisms.

Figs. 59 A, B, C, and D show typical distortion curves of prisms as determined by the lens-measuring apparatus illustrating the effects of the first four factors. Since these curves approximate straight lines it is

simple to write the distortion equation for a given prism in a given position,[16] in terms of the rate of change of the distortion, or the slope of its curve. This is denoted by h and is peculiarly related to the Hering-Hillebrand horopter deviation. The ordinate intersect of the curve gives the overall magnification value.

The variations in the longitudinal horopter produced by the use of prisms before the eyes as determined by the apparent frontal plane or grid-nonius horopter, will be considered.

If two prisms are used it must be remembered that the shape and position of the horopter depends upon the unbalanced distortion of the ocular images in the two eyes, hence the effect produced will be the sum of the effects of each prism when placed before the eyes.[16] Since the above holds strictly only for symmetrical convergence of the eyes, prisms of unequal prism power will not be considered (cf. preceding section). Likewise, placing a pair of prisms before the eyes so that the bases are in the same direction, will not be considered. In general, when prisms are used before the eyes, the change in the apparent frontal plane or the grid-nonius horopter is an increase or a decrease in the curvature. This indicates a change in the Hering-Hillebrand horopter deviation value. Its direction and magnitude is dependent upon the manner in which the prisms are placed before the eyes (i.e., base in or base out). If the prisms are placed base-in before both eyes, the effect is to make the horopter more concave to the observer, i.e., to increase the Hering-Hillebrand deviation.[44,49] Prisms placed base-out have the opposite effect, that is the horopter is made convex to the observer, the Hering-Hillebrand deviation is decreased. Theoretically the quantitative change in the Hering-Hillebrand deviation can be predicted if the slope of the distortion curves of the two prisms h_1 and h_2 and the Hering-Hillebrand constant H_0 for the unaided eye are known. Then, within the precision of measurements, the resultant deviation which specifies the shape of the horopter when the prisms are worn, should be

$$H = H_0 \mp (h_1 + h_2)^{16}$$

the negative and positive signs being used for the cases of prisms placed base-in and base-out O.U., respectively. But certain measurements of the longitudinal horopter made with lenses which in any way affect the accommodation or the convergence show that this equation does not hold. This is probably due to a change that has occurred in H_0. It is clear that if the overall magnification values of the two prisms

[49] Helmholtz found this phenomenon, reference 36, p. 324.

are not equal the horopter will be oblique, i.e., rotated about the point of fixation by the amount of difference between the two values. Furthermore, it is clear that if the distortionless prisms[50] ($h_1 = h_2 = 0$) are used, no change occurs in the shape of the horopter. Table 18 and Fig. 60 show experimental data for Mr. Ames with the apparent frontal plane illustrating the effect of prisms. It should be noted that if distortionless prisms are used although the shape of the horopter is not changed, (save in respect to the exception taken above) its position relative to the fixation may be altered due to an induced change in fixation disparity resulting from the prism power as has been described.

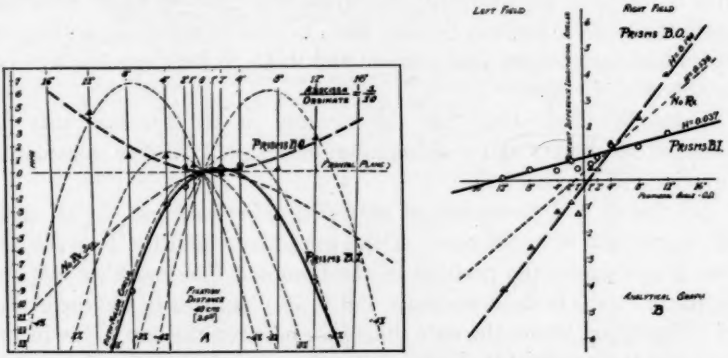


FIG. 60. *The spatial and analytic graphs of the apparent frontal plane when prisms base-in and base-out have been placed before the eyes. Data of A. Ames, Jr.*

(3) Typical changes in the shape and position of the horopter, produced by the use of power lenses

Many complex changes in the horopter may result from the use of power lenses before the eyes. In fact, practically all glasses that are prescribed as corrections affect the horopter to a greater or lesser degree. Aside from the change in the position of the horopter due to a fixation disparity resulting from phoria induced by plus or minus power, which has been described, changes result from the dioptric

[50] Because the ordinary prism then has a certain amount of distortion, it was necessary to design a set of prisms which gave no distortion. This can be accomplished by using the proper base curve. Such prisms still have a residual overall percent magnification in the meridian of the prism. They also have a different magnification in the horizontal and vertical meridians. To have the same magnification in both meridians, the base curve of the vertical meridian must be different from that of the horizontal. By combining such prisms with lenses which reduce the overall size, the combination will not only be distortionless for a given fixation distance, but it will have a zero magnification effect.

effect of the lenses which alter the projection fan of one eye relative to that of the other. The most marked effect is produced when lenses of different power are put before the two eyes. Due to the difference in size of the images produced, the same effect results as when a "size" lens is put before one eye. The dioptric effect of lenses insofar as distortion is concerned depends not only upon their inherent characteristics, but also upon their position relative to the eye. It follows that the alteration of the projection fan of one eye relative to that of the other may result not only when lenses of different power are put before each eye, but also when lenses of equal power are used if their positions relative to the eyes are different. The distortion effect of lenses resulting from the positioning may be due, first, to their distance from the eye; second, to their lateral displacement and third, to their rotation relative to the axis of vision.

Examples illustrating that the position of the horopter may be changed by the use and positioning of lenses prescribed in ophthalmic practice will be considered.

(*a*) *Lenses of different optical properties before each eye.* In all cases the correction of a difference of the refractive condition between the two eyes changes the position of the horopter. The most evident example of this is in anisometropia, but it also happens in the correction of astigmatism where the only difference between the correction in the two eyes is the axis of the astigmatism. In general, the exact amount of change in the size of the dioptric image in the eye when a power lens is used before the eye cannot be exactly predicted, for it depends to what extent the eye absorbs the power of that lens, i.e., to what extent a corresponding change in the accommodative state occurs. The latter is affected by the fixation distance and the accommodative state of the eyes. Data have been computed for this change in size, both with complete absorption and no absorption of the power of the lens. These are approximate only. They are shown in Table 19. Evidence obtained from experimental work on the horopter gives approximately a 2 percent difference in size of ocular images for a difference in power of 1 diopter between the eyes for a fixation distance of 400 mm. There is also a distortion effect from power lenses varying with power, base curve, and distance from the eye. See Fig. 61 A, B, C.

(*b*) *Lenses decentered with respect to visual axis.* Changes in the position of the horopter are produced in all cases where the optical axes of the corrective lenses are displaced with respect to the visual axis (except where with similar lenses, they are displaced the same amount in the

same direction). The most common example of this is where the power lenses are decentered to give a prismatic effect. The resulting distortion is similar to that produced by prisms described above, and is governed by the same laws. The amount of change in the size and distortion of the fans produced by decentering power lenses has been determined by measuring on the lens bench various lenses with different amounts of decentering. Typical examples of such distortion are given in Fig. 61, D and F. This distortion, which is small with weak lenses, increases rapidly with increase of power.

TABLE 19. *Table showing the change in the size of the dioptric image of an object placed at 6.1 m, when different power lenses are placed at the anterior focus of one eye.*

Dioptric power of lens	When the eye remains accommodated for 6.1 m	When the accommodation of the eye changes to absorb the power of the lens
+10.0	26.8%	22.5%
+7.5	18.8	15.7
+5.0	11.8	9.8
+4.0	9.2	7.6
+3.0	6.8	5.5
+2.0	4.4	3.6
+1.0	2.2	1.7
0.0	0.0	0.0
−1.0	−2.1	−1.8
−2.0	−4.0	−3.4
−3.0	−6.0	−5.0
−4.0	−7.8	−6.5
−5.0	−9.5	−7.9
−7.5	−13.7	−11.4
−10.0	−17.4	−14.5

(*c*) *Lenses rotated with respect to the visual axis.* Generally lenses are so positioned relative to the head that it is only when the eyes are in their primary position that their lines of sight pass through the center of the lenses normal to their surfaces. With near vision, when the eyes converge and look down, the lenses are both decentered and rotated relative to the lines of sight. In general this is even found in glasses designed for near use and in bifocals. As a result, in all cases where glasses are worn, the position of the horopter will be more or less affected be this factor.

If the power of the corrective glasses is small, the change in the horopter is slight and can be neglected, but with higher powers it may become considerable. The amount of change in the size and distortion

of the image produced by the tipping of lenses has been determined by measuring on the lens bench various lenses with different amounts of tipping. Typical examples are shown in Fig. 61, E and F. If the glasses have been properly placed before the eyes when in their primary position in reading, the angles between the visual axes and the lenses are about 4.5°, and the decentering in each is about 2 mm in and 6 mm down.

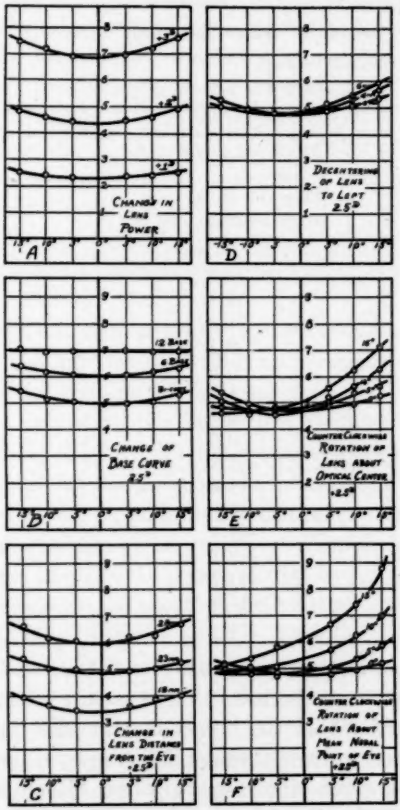

A
CHANGE IN LENS POWER
D
DECENTERING OF LENS TO LEFT 2.5°
B
CHANGE OF BASE CURVE 2.5°
E
COUNTER CLOCKWISE ROTATION OF LENS ABOUT OPTICAL CENTER +2.5°
C
CHANGE IN LENS DISTANCE FROM THE EYE -2.5°
F
COUNTER CLOCKWISE ROTATION OF LENS ABOUT MEAN NODAL POINT OF EYE +2.5°

FIG. 61. *Typical distortion curves of power lenses.*

Since different combinations and magnitudes of the previously mentioned factors will be effective when glasses are placed before the eyes, the resultant changes on the shape of the horopter are difficult to predict exactly. However, considerable information can be obtained concerning the effect of glasses being worn, if the horopter (or apparent

frontal plane) is determined both with and without the glasses. The nature and magnitude of the effective unbalanced distortion due to particular lenses placed before the eyes is given by the deviation of the analytical graph for the horopter (apparent frontal plane also) when the lenses are worn, from that found with the unaided eyes. If no effective unbalanced distortion or size difference is produced by a given correction, the two curves will be the same.

VII. SUMMARY

The important concepts and deductions of the foregoing sections may be summarized as follows:

(1) The concept that associates specific subjective directional values with specific cortical centers is fundamental to the understanding of the relationship between subjective and objective space and directional values.

(2) The way in which specific cortical centers are represented on the retina of each eye together with their dioptric characteristics determine the horopter and the relative size and shape of the ocular images.

(3) The optical projections of the retinal points represent the effective distribution of the spatial values associated with the specific cortical centers to which the retinal points are connected.

(4) Lines drawn from object points on the horopter in the visual field, through the perspectival centers of the two eyes placed at the mean nodal points of the schematic eye, approximate the actual optical projections of the corresponding retinal points within experimental error. The angles between these lines provide a basis for quantitative measurement. A series of such optical projections of retinal elements associated with certain directional values constitutes projection fans. (a) Where these lines are the optical projections of corresponding retinal points the two fans provide a means of representing the objective relationship of corresponding retinal points. (b) Where these lines are the optical projection of a series of equal subjective space values (as projected from each eye) the fans provide a means of representing the objective relationship of a series of retinal points separated by equal subjective space values.

(5) The effective monocular distortion of the ocular image in each eye can be ascertained by determining such objective angles (4b) between a series of points adjusted in space so as to give the subjective impression of being equally spaced. (Multiple partition experiments.)

(6) The effective relative binocular distortion (in the horizontal

meridian) is expressed in the shape of the longitudinal horopter in the horizontal meridian. Any overall inequality in the images or of the projected spatial values (i.e., size) is manifested in the degree of rotation of the longitudinal horopter about a vertical axis at the fixation point. The magnitude of the relative distortion of the ocular images or the asymmetric distribution of the projected spatial values is expressed quantitatively by the Hering-Hillebrand deviation. This is obtained from so-called analytical graphs in which the empirical data are expressed and plotted as the percent difference between the angles made by corresponding lines of the two projection fans to a given point and the fixation lines against their angle of obliquity. The data so plotted tend to approximate a straight line, the slope of which is a measure of the Hering-Hillebrand deviation value and is designated as H. Such analytical graphs give a quantitative measure of the difference in the size of the ocular images and also the sum of the asymmetrical distribution of corresponding projection lines.

(7) The effective monocular distortions (5) of the ocular images in the two eyes are quantitatively related to the shape of the horopter. The algebraic sum of the Kundt or the Münsterberg conditions in each eye is equal to the effective binocular distortion as determined by the shape of the horopter.

(8) Determinations of the longitudinal horopter both by Tschermak's binocular nonius method and by a modification of it designated as the grid-nonius method, indicate that the criterion of the apparent frontal plane is not correct for the determination of the true horopter.

(9) The displacement of the true longitudinal horopter relative to the fixation point indicates that the images of the point of fixation fall on disparate elements (not corresponding points). This fixation disparity is shown to be the same phenomenon previously described as retinal slip and is thus correlated with phoria. The horopter is found to lie behind or in front of the point of fixation depending upon whether the phoria is exophoric or esophoric, respectively.

(10) The subjective direction of disparate images is that associated with a near pair of corresponding points. In general this phenomenon occurs for any point object not lying on the horopter and is evidenced in the difference of the apparent direction of such objects when seen binocularly and monocularly. The phenomenon of Panum's limiting case is shown to be due to fixation disparity.

(11) The relative effective projections of corresponding points are shown to change with fixation distance, asymmetrical convergence,

and from other unknown causes, as indicated by the records of certain clinical patients. These experiments would indicate either a change of the projection of corresponding points or a change in the visual directions associated with corresponding points.

(12) In general it was shown that the effect of ophthalmic lenses upon the shape of the horopter is that resulting from the residual unbalanced distortion between the lenses placed before the eyes.

VIII. CONCLUSIONS

The general conclusions of the paper may be listed as follows:

(1) The studies indicate that the relationship between subjective and objective space and directional values are subject to more exact measurement than had heretofore been recognized.

(2) They indicate that a mathematical comprehension and formulation of the empirical longitudinal horopter data are possible; first, through an improved method of obtaining, handling, and analyzing the empirical data; second, through the elimination of those factors that can be neglected; and third, through empirically checking the magnitude of changes introduced by known factors.

(3) It would appear that the fundamental hypothesis of specific directional values associated with specific brain centers is placed on a firmer basis.

(4) It would appear that certain complex phenomena associated with binocular vision heretofore calling for psychological interpretation have been given a physiological explanation.

THE RELATION BETWEEN THE COMPOSITION AND THE DENSITY AND OPTICAL PROPERTIES OF GLASS. I. THE SODA-LIME-SILICA GLASSES

By G. W. Morey and H. E. Merwin
[Geophysical Laboratory, Washington, D. C.] Received June 27, 1932

This paper is the first of a series dealing with an investigation undertaken to provide a more extensive and accurate body of data on the relation between the composition and the density and optical properties of glasses. Parts of several ternary and quaternary systems have been studied, and in two systems the study has been extended to the extreme composition limits at which the inherent tendency to crystallize makes impossible the preparation of glasses even by laboratory methods. Excepting some recent work, the information in the literature[1] deals almost exclusively with the restricted composition range possible in commercial glasses, and the investigations usually have been made with a technical purpose in view.

Besides the work reported in this paper, there has been carried out a study of the phase equilibrium relationships, which have been discussed in several papers.[2] In Fig. 1 are indicated the compositions of the various compounds which are formed in the system, as well as the fields of these compounds.

In planning our work it was deemed desirable to make, as nearly as possible, a complete study of the effect of each component on the optical properties and density. Even in the phase equilibrium studies it was not possible to work with mixtures approaching in composition the alkali or alkali earth oxides, both because of the difficulty in working with such high melting and corrosive substances, and because of the impossibility of obtaining the mixtures in the glassy condition. However, we have studied a much wider composition range than has been known hitherto, especially in the ternary systems, Na_2O–CaO–SiO_2 and K_2O–CaO–SiO_2, in which the glasses studied are well distributed over the diagram from mixtures intermediate between the orthosilicate and metasilicate ratios to pure silica. The distribution of glasses in the system, Na_2O–CaO–SiO_2, is shown in Fig. 1, in which is plotted the composition of each of the

[1] A comparison of our results with those of other recent investigators begins on page 659, and a brief summary of the older work in a footnote on the same page.

[2] G. W. Morey and N. L. Bowen, J. Soc. Glass Tech., *8*, p. 226; 1925: G. W. Morey, J. Am. Ceram. Soc., *13*, p. 683; 1930.

glasses studied. In addition, numerous glasses in the ternary systems formed by magnesia, alumina, ferrous or ferric iron, titania and boric oxide, with SiO_2 and Na_2O, have been studied, as well as numerous quaternary glasses containing MgO, Al_2O_3, or B_2O_3 together with Na_2O, CaO, and SiO_2, and many miscellaneous glasses of unusual composition. In this paper are given or indicated the methods of study used through-

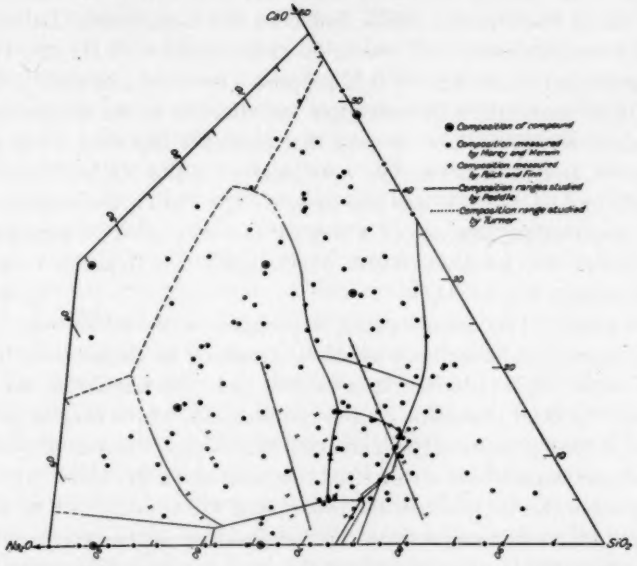


FIG. 1. *The compositions of the glasses measured by Morey and Merwin, and by other observers, plotted on a diagram showing the compositions of the compounds formed in the ternary system, $Na_2O-CaO-SiO_2$, and the boundaries of the fields of these compounds.*

out the work, the results for the glasses of the system, Na_2O–CaO–SiO_2, and such deductions as are pertinent to the system under consideration. The discussion of the results as a whole will be deferred until the final paper of the series. The work reported on by the authors at the 1925 meeting of the Optical Society of America,[3] and later reproduced without details in the International Critical Tables[4] is included in the present paper, together with more than an equal number of additional glasses.

[3] G. W. Morey and H. E. Merwin, J. Opt. Soc. Am., *11*, p. 125; 1925.

[4] G. W. Morey, International Critical Tables, *2*, pp. 92–102; McGraw-Hill Book Co., Inc. New York, 1927.

Preparation of the Glasses

The glasses were made from SiO_2, in the form of quartz, precipitated $CaCO_3$, and Na_2CO_3, by melting the requisite quantities in platinum crucibles, mixing by powdering and sieving, and remelting until homogeneous. The raw materials were of the highest obtainable purity. Two samples of quartz were used. The earlier glasses, comprising about two-thirds of the whole, and fairly uniformly distributed in composition, were made from quartz which had been the Geophysical Laboratory stock for some years, and which, on evaporation with HF and H_2SO_4, and ignition, left a residue of 0.07 percent. The second lot gave a residue of 0.04 percent, and a spectroscopic examination of the residue by our colleague, Dr. E. G. Zies, showed that probably less than 5 percent of it was Fe, and the other metals were largely Ca and Na, with relatively less Mg and Al. Some P was also present. The $CaCO_3$ was an unusually pure preparation, analysis of which by Dr. Zies gave, in percentages, SiO_2, 0.002, $Fe_2O_3+Al_2O_3$, 0.006, MgO, 0.003. The Na_2CO_3 contained 0.009 percent $Fe_2O_3+Al_2O_3$.

While most of the glasses could be prepared without difficulty, some of the extreme compositions could be obtained as glasses only by the most careful manipulation. It is evident that this should be the case, because the chief limitation on the compositions which may be used as glasses is that placed by the increasing tendency toward devitrification as mixtures depart from a restricted composition range; and it is evident that glasses can be made in the laboratory which could not be manufactured on a commercial scale. The preparation of the extreme glasses will be reverted to after discussing the usual method of preparation.

With most of the glasses the only difficulty is that of mechanical loss as the result of the too rapid decomposition of Na_2CO_3, a difficulty easily overcome by carrying out the initial decomposition at such a temperature that the mass merely sinters, enabling the CO_2 to escape from a porous mass rather than from a thick viscous frothy mass. About the first third of the glasses were made in a gas-fired furnace, weighing out the ingredients in the proper proportion, mixing, and adding the charge in portions as it melted down. All such glasses were analyzed. The melting process was repeated, with intermediate crushing and sieving, until the glass was homogeneous, as shown by examination of the powder under the microscope, using an immersion liquid matching it in refractive index. By this method a variation in refractive index amounting to 1 to 2 units in the third decimal place can be detected readily. The final test came when the prisms were ground, when, if striae showed,

the glasses were remelted in an electric furnace, account being taken of the change in weight.

The later glasses were made in a platinum-wound electric furnace, which could be heated to 1600°, and of such a size that it would take a 25 cc platinum crucible. The difficulty of mechanical loss was easily avoided by putting the platinum crucible containing the powdered and mixed ingredients into a cool furnace which would require some hours, usually over night, to come to about 1000°, after which it could be quickly heated to 1300 to 1400°, at which temperature most of the glasses were fined. After the initial heating, the glasses were powdered in a specially hardened steel mortar to pass 66-mesh bolting cloth, then remelted, and this process of powdering and remelting continued until no inhomogeneity could be detected in the powdered glass when examined under the microscope as described above. After this the final heating was made in a clean platinum crucible. The composition of the glasses so made was established by careful control throughout the melting process. The platinum crucible in which the meltings were made was weighed before and after each heating and each pulverizing operation, so that, knowing the amounts of the nonvolatile oxides, CaO and SiO_2, the exact composition of the glass could be calculated. The sources of error are (1) the variation in weight of the platinum crucible during the melting operation, either by loss of platinum, an unimportant item, or by material sticking to the outside of the crucible, a source of error difficult to avoid at high temperatures where the best of refractories tends to stick to platinum. This was finally obviated by resting the crucible on a flat polished plate cut from a corundum crystal. The melting record for five platinum crucibles, used for forty glasses, with intermediate cleansing by fusion with Na_2CO_3, showed a total loss in weight of 10.7 mg, or an average loss (including the loss on Na_2CO_3 fusion) of only 0.25 mg per glass, a negligible amount. (2) The material lost during the powdering operation may not be of exactly the same composition as the melt as a whole. The loss was rarely 0.1 g, and usually was one-half to one-third this, hence, to make an appreciable error the portion lost would have to be very different from the rest; but the glass at the first pulverization was one which already had reached some degree of homogeneity during fining. (3) The material lost on heating might not be Na_2O alone, but also some CaO or SiO_2. This is not regarded as probable; only on the first heating is there danger of mechanical loss, and especial care was taken to avoid loss during this period. Usually a factor was used to convert $CaCO_3$ to CaO, and the CaO content is of

course dependent on this factor. It was usually determined by heating the $CaCO_3$ for some melt high in CaO until a constant weight was obtained, and this factor was frequently checked. The SiO_2 was dried at 500°, and all ingredients were stored in a desiccator over KOH. None of these errors, however, is of great moment, and it is believed that the compositions of the glasses made in this way are better known than would be possible by chemical analysis.

To make more clear the exact procedure, the log of a typical melting is reproduced in Table 1; the glass is No. 97 of Table 3, calculated to have the composition of the compound, $Na_2O \cdot 3CaO \cdot 6SiO$: Na_2O, 10.50 percent; CaO, 28.45 percent; SiO_2, 61.05 percent.

TABLE 1. *Log of a typical glass melting*

	Weight	Difference
Platinum crucible	15.1035	
Plus $CaCO_3$	20.1959	5.0924
Plus SiO_2	26.3016	6.1057
Plus Na_2CO_3	28.4109	2.1093
After 1st fining	25.0942	
After passing 66 mesh	25.0231	loss=0.0711
After 2nd fining	25.0218	
After passing 66 mesh	24.9677	loss=0.0541
After 3rd fining	24.9665	

The glass, powdered to pass 66-mesh bolting cloth, then was transferred to a clean platinum crucible, and fined, during which fining the glass lost 0.7 mg. The final composition is that calculated for No. 97, Table 3.

The above procedure was followed in all cases insofar as experimental difficulties did not interfere. Glasses low in Na_2O, i.e., those near the SiO_2 apex and near the more siliceous portion of the $CaO \cdot SiO_2$ side of the component triangle, were difficult to cool without becoming milky from separation of crystalline silica, and the high temperature required to melt them resulted in excessive loss of Na_2O. To obtain them entirely free from devitrification it was necessary to drop the crucible containing the molten material directly from the furnace into cold mercury. This was effected by an electrical device, and the tiny crucible used was wrapped in thin platinum foil in such a manner that mercury could not get inside the wrappings. Some of the glasses intermediate between $Na_2O \cdot SiO_2$ and $CaO \cdot SiO_2$ were difficult to obtain in large melts without devitrification, but smaller melts were easily prepared by quickly plunging the lower half of the platinum crucible into either

water or mercury, taking care not to get any of the liquid inside the crucible. Glasses high in both Na_2O and CaO were difficult to melt because of their high melting temperature, the corrosive nature of the highly alkaline glasses, the rapid loss of alkali, which caused a composition gradient to be set up quickly, and, in particular, because of the difficulty in cooling them without crystallization. Melts were made so rich in Na_2O and CaO that they could not be obtained as glass even by the most rapid cooling, in lots of only a few milligrams.

CHEMICAL ANALYSES

While the compositions of most of the later glasses were satisfactorily established by careful synthesis,[5] analyses were required for all of the earlier melts, as well as for many of the later ones in which the controlled synthesis procedure either could not be followed or was interrupted by some mishap. Also, in several cases analyses were made of glasses in strategic positions even though their compositions had been established by controlled synthesis. In the analytical procedures the methods recommended by Hillebrand and Lundell[6] were used as guides. In many cases, the glass was decomposed by fusion with Na_2CO_3, after which CaO and SiO_2 were determined directly. In other cases, the glass was decomposed by evaporation with HF and $HClO_4$, after which CaO was weighed as such, Na_2O as Na_2SO_4. SiO_2, when determined directly, was always separated by two evaporations with HCl, with intervening filtration. CaO was separated as oxalate, making a double precipitation. As a rule, Fe_2O_3 and Al_2O_3 were not determined directly, hence would appear as impurities in the CaO, raising that percentage slightly. They were determined directly in a few cases, and found to be of the order of the impurities in the quartz. Some glasses in the binary system, Na_2O–SiO_2, were analyzed by evaporation with HF, and weighing the Na_2O as Na_2SiF_6, after drying at 125°; and in other cases the Na_2SiF_6 was changed to Na_2SO_4. In Table 3, in which the compositions are given, those percentages in parenthesis refer to constituents determined by difference, hence to analyzed glasses. The rest of the compo-

[5] It should be emphasized that there is a difference between such controlled syntheses as we have described, in which the weight taken after each operation is a direct check on the composition and proportion of the ingredients, and those syntheses in which the calculated quantities of the materials are mixed and melted without further check. In the latter case, chemical analysis is necessary to establish the composition. A sufficient check may sometimes be made by measuring throughout a series some property which changes with composition.

[6] W. F. Hillebrand and G. E. F. Lundell, "Applied Inorganic Analysis," John Wiley & Sons, New York, 1929.

sitions, in which the percentages add up to 100, and are not enclosed in parentheses, were established by controlled synthesis. Further information in regard to the composition in some cases is given in the notes.

PHYSICAL MEASUREMENTS

The accuracy of the physical measurements was conditioned by factors other than the methods of making the measurements themselves. Except in special cases there is no reason for carrying the physical measurements to a higher degree of precision than that corresponding to the homogeneity of the glass, or the precision with which the composition is known, or to the definiteness of the annealing procedure.

The density determinations were made by the flotation method, using a standardized Westphal balance. The balance was adjusted with a clear quartz test piece, $d_4^{20°}$ 2.649$_3$. For non-hygroscopic glasses, Klein's solution (cadmium borotungstate) commonly was used, while for hygroscopic glasses *s*-tetrabromethane with acetone, or methylene iodide with benzene, was used. Single fragments of glass weighing about 0.5 g were used when available, but bubbly glasses and those partly milky had to be broken smaller, so that several fragments were used together. Duplicate measurements in all but very unfavorable cases agreed within ± 0.001.

For refractive index, the minimum deviation method was used with a goniometer equally suitable for prisms 2 by 4 mm or even smaller, and for large prisms. Prisms with angle of 70–80° were ground and polished by hand without attempting to produce very perfect surfaces. The surfaces were covered with carefully selected strips of microscope cover glass, held on by films of nonvolatile liquid of approximately the same refractive index as the prism. To prevent movement of the covers the liquid film was so thin that they were not floated. The original surfaces were good enough to permit the approximate measurement of the prism angle, and of refractive index if desired, so that the parallelism of the glass plates to the prism faces could be sufficiently tested. Divergence of 30′ could be tolerated[7] when differences in index between prism and liquid were less than 0.040, but careful exclusion of dust from the capillary space kept the divergences within a few minutes of arc.

[7] As a first approximation, the index as measured on the composite prism may be corrected by the following formula, which gives the proper algebraic sign in each of the four possible cases:— $[(\delta_c-\delta_i)/\delta_c](n_i-n_l)$, where δ_i=angle of inner prism, δ_c=angle of composite prism, n_i=refractive index of the inner prism, n_l=refractive index of the liquid.

Homogeneity of the prism was tested by observing any widening or multiplication of the signal, and by adjustable diaphragms at the incident surface of the prism. Badly striated glasses were powdered and remelted. In a few glasses, very high in Na_2O, horizontal inhomogeneities caused by volatilization produced striae in a short remelt, while in a long remelt new inhomogeneities appeared. In most cases the sodium lines from the glass of the hydrogen tube were the source of light, but with striated glasses these lines sometimes did not stand out from the yellow lines of the hydrogen spectrum, in which cases values were deduced from the helium or mercury yellow lines.

The dispersions of several glasses very high in either soda or lime were determined[8] for a wide range of wave lengths, but for most of the glasses only $(F-C)$ was measured. Assuming that the prism angle was correctly measured, the probable error in each C and F rarely exceeded ± 0.0001. Account was taken of the effect of the wedge of liquid under the cover slip.[9] In a few cases of striated glasses the F-line was indistinct and F was interpolated from measurements on Hg_{4916} and Hg_{4358}. We have always found Hg_{4916} visually very much brighter than Hg_{4960}. Measurements of dispersion including the latter are open to question.[10]

ANNEALING

It was desired to make our measurements on glasses reproducible and comparable, free from the influence of such variables as might be introduced by heat treatment. It has long been believed that mechanical strain appreciably affects the optical properties and density of glass,[11] and that the effects can be removed by heat treatment, but it is not evident in what manner a system of balanced stresses such as exist in strained glass could affect the bulk density or refractive index. Whether or not the effect of heat treatment consisted solely in the removal of mechanical strain seemed an open question, but several investigators[12]

[8] The shape of the curve for high soda glasses is like that of light flints; for high lime glasses, like that of barium silicate crowns.

[9] The formula $[(\delta_c-\delta_4)/\delta_4][F-C)_4-(F-C)_1]$ may be used as previously described.

[10] See International Critical Tables, *5*, p. 299; and *7*, 102–103.

[11] H. Hovestadt, *Jena Glass*, trans. by J. D. and Alice Everett. Macmillan and Co., New York, 1902.

A. Q. Tool and J. Valasek, Bureau of Standards Scientific Paper No. 358, 1920.

C. G. Peters and C. H. Cragoe, Bureau of Standards Scientific Paper No. 392, 1920.

S. English and W. E. S. Turner, J. Soc. Glass Tech., *4*, p. 153; 1920.

[12] S. English and W. E. S. Turner, J. Soc. Glass Tech., *4*, p. 153; 1920. A. Q. Tool and C. D. Eichlin, J. Opt. Soc. Am., *8*, p. 419; 1924. A. Q. Tool and C. D. Eichlin, J. Am. Ceram. Soc., *8*, p. 1; 1925. A. Q. Tool and E. E. Hill, J. Soc. Glass Tech., *9*, p. 185; 1925. C. G. Peters, Bur. Standards Scientific Paper No. 521, 1926.

have made it probable that the effect is not only to remove the mechanical strain, but also to produce other definite changes in the glass. Our experiments have led to similar conclusions. It was found that over wide ranges of composition, density changes produced by heating over night at 450°–500° were about equal. As work progressed, other heat treatments were found to give greater changes, especially in glasses of extreme composition. In some early tests the amount of surface volatilization was determined by careful weighings, but in no case was the amount significant.

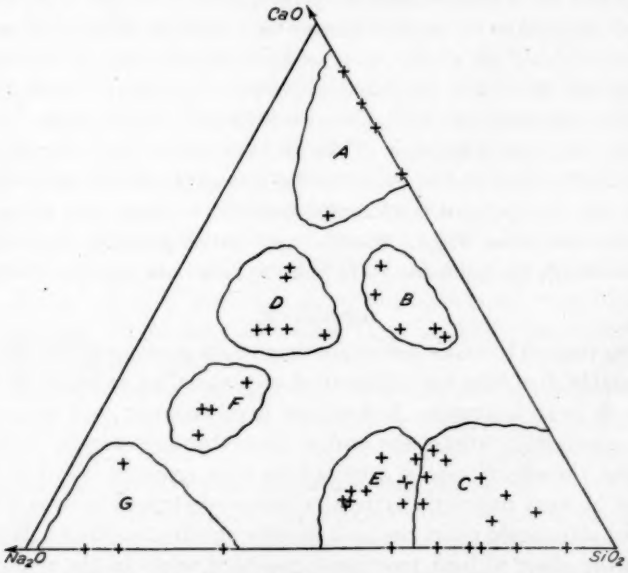


FIG. 2. *The compositions of the glasses used in the study of annealing.*

The compositions studied most thoroughly are shown in Fig. 2, and are given in Table 3, and the density changes are given in Table 2 a, b, and c. For these experiments pieces weighing about 0.3 g were used. Table 2a gives the results of heatings of 5 to 15 minutes at increasing temperatures, made to determine where the glasses begin to show "fire-polish," by incipient flowage at the surface, or where they begin to crystallize without such flowage. These samples were cooled rapidly by dropping from the furnace into the air. The densities after heating at the highest temperatures were the lowest, often 0.004 to 0.007 less than for samples prepared as already described. Glasses of area *A*, Fig. 2, with

over 40 percent CaO and little Na_2O, gave these low densities when cooled from about 900°, at which temperature they crystallized without definite surface flowage. Glasses of area *G*, Fig. 2, containing 40 percent

TABLE 2a. *Density changes near the softening temperature. Differences (heat treated—original)* $\times 10^5$.

Glass Nos. and area, Fig. 2	600°		650°		700°	760°	800°	830°	875°	915°	450° to 500°
	15 m.	45 m.	15 m.	17 hrs. (3 hrs.)	15 m.	5 m.	15 m.	10 m.	5 m.	5 m.	17 hrs.
152		+2							*n*	*n, d*	+ 6
139			+ 8								+ 6
125 *A*			+ 1	+6		+2	0	0	0, *n*	*n, d*	+ 5
107			+ 2	+9[1]		0, *r*					+ 3
124		+5	+ 4	(+4)		−2		−7	*n**	*n, d*	+ 3
97		+3	+ 1	(+2)		−2, *r*					+ 6
82			<+11								+16
63 *B*			+11								+12
50*a*	+5	+6 [+9][4]			−1	*n*	−4, *r**				+ 3
9			+ 1		−2		−2, *r*				+ 6
5			<+ 4								+ 7
8 *C*			0		0, *r*						+ 6
4			− 7								+ 2
127			*d*								+13
104	0		*n*		−4, *r*						+ 4
119 *D*		0									+ 6
126*a*		+ 1	*d*								>+ 5
58	−3, *n*		−3, *r*[3]								+ 7
47 *E*			+10								+16
60			−4, *r*								+ 3
122*a*		0	*d*								+ 2
131 *F*		*d*[2]									>+ 5
146 *G*		*d*									+ 3

d, devitrified or decidedly opalescent; *n*, no evidence of surface flowage; *r*, corners rounded or surface showing polish due to slight flowage; * slight opalescence; < samples which before this heating had a maximal density (as indicated by the figures), are less dense after heating.

[1] Density increased 0.006 in 3 hours.

[2] Also was devitrified after 7 days at 383°.

[3] Rounding visible after 5 minutes.

[4] Density was 0.009 higher after 21 hours at 600°, but a similar heating did not change the density of a piece which had previously acquired a high density.

TABLE 2b. *Density changes in the annealing ranges. Differences (heat treated—original)*$\times 10^3$.

Glass Nos. and area, Fig. 2	555°			500°			460°				383° or (400°)				325°		277°	230°
	15 m.	3 hrs.	24 hrs.	1 hr.	17 hrs.	41 hrs.	2 hrs. or (6 hrs.)	17 hrs.	38 hrs.	3½ da.	6 hrs.	2 da.	3½ da.	7 da.	2 da.	123 da.	22 da.	109 da.
139 *A*		+ 6						+ 6								+ 9[a]		
98 *B*	+12							+11										
82 *B*	+12	+14						+16[f]								+11[a]		
63 *B*	+ 7	+ 9						+12[f]								+11[a]		
50*a* *B*	+ 3		+9	+4	+9	+9	+2	+ 3	+4	+ 5	(+1)	(+1)	(+ 2)	(+ 2)				
5 *C*[i]	+ 3	+ 6						+ 7										
127 *D*	+ 4	+11						+13								+10		(0)[c]
104 *D*	+ 1		+5	+4	+7	+7	+2	+ 4	+5	+ 5	(+1)	(+1)	(+ 2)	(+ 4)				
126*a* *D*							(+5)	+ 6						+ 6	+ 5			
131*a* *D*							(+5)	+ 7						+ 8	+ 7			
48 *E*								+ 7					+ 6	+ 8	0			+ 1
58 *E*	− 1		−3	+5	+5	+5	+ 6	+ 7	+8	+10	(+4)	(+5)	+(6)	(+ 6)[e]				
57 *E*								+ 6					+ 8	+ 9				
47 *E*	+11	+11						+16							+ 3	+15		
53 *E*								+ 6					+10	+10				
122*a* *F*								+ 2					+ 4	+ 4				
133 *F*								+18					+15		+ 6			+ 3
134*a* *F*							(+2)	+ 5						+11	+ 5			
131 *F*							(+5)						+13	devit.	+ 7			
146 *G*							[+11][h]	+ 3			[+10][g]	+7		+ 8	+16		+15[d]	+14[b]
98*a* *G*																		

[a] Density at 650° remained unchanged.
[b] Increased 10 in 47 days at 230°.
[c] Refers to No. 131*c* having similar composition.
[d] Changed 13 in 3 days.
[e] A piece which had gained 10 at 460° did not change, but an unannealed piece gained 6.
[f] Temperature 450°–500°.
[g] 16 hours at 375°.
[h] 15 hours at 460°.
[i] See Table 2c.

or more of Na_2O and little CaO, devitrify or show surface flowage after a few minutes heating at 500°, or possibly lower. Many compositions near the side, CaO–SiO_2, and containing more than 75 percent SiO_2 cannot be made into homogeneous glasses because of formation of immiscible liquids,[13] or of crystallization, or of the presence of fine bubbles. The highest silica glasses of Table 2a in areas *B* and *C* showed evidence of surface flowage at 800° in 15 minutes, and when quenched were 0.002 –0.004 less dense than the original glasses. Beyond the area *ADFG* the

TABLE 2c. *Density changes in the annealing ranges. Differences (heat treated—original)*$\times 10^3$.

Glass Nos. and area, Fig. 2		Days at 395°				
		2	6	12	21	29
1		2				
1*a*		3[a]	5	5		
2*a*		4[a]	5	5		
3*x*[b]		9	9			
3*a*	*C*	2	3	3		
3*b*		5	5			
5*a*		7	8	9	10	
9*b*		7	9	9		
10			0	2	2	
11		5	6	6		
13		8	9	10	11	11
17	*E*	4	5	8	8	
20		3[a]	5	7	8	8
21		7	0	0		

[a] Gain in 15 hrs. at 460°.

[b] A 0.3-gram charge in a tight platinum container quenched from 1450° in mercury, $d=2.320$. A similar charge cooled in air, $d=2.333$. A 5-gram charge cooled on a metal plate, $d=2.320$–2.335 for small fragments. Both of the last looked clear, but when suspended in a heavy liquid and illuminated from the side milkiness appeared. The crystals were not visible with the microscope.

glasses crystallize so readily that no studies of minimum density could be carried out.

As to maximal density, a general, but not in all cases detailed, agreement was found. The detailed results of these studies are assembled in Table 2, b and c. At about 200–300° below the temperature of incipient flowage a glass may be brought to a maximal density within the relatively short time of a few hours or days. From this temperature to that

[13] J. W. Greig, Am. J. Sci., *13*, p. 147, 1927.

of incipient flowage the ultimate density diminishes regardless of the time of heating; somewhat below this temperature maximal density may be reached with long-continued heating. The lime-silica glasses in area *A* required several hours at 600–650° to complete this density change, which amounted to from 0.004 to 0.009. The total density differences between maximal and minimal values were 0.006 to 0.011. In area *B* the maximal density was attained at 460° to 600°, and the total changes were 0.013–0.016. In area *C* the temperatures were about as in *B* but the total changes were only 0.008–9. In areas *D* and *E* the minimal densities were found near 600°, the maximal near 400°, and the total change was about 0.015. Especially near areas *F* and *G*, glasses may tend to become opalescent, owing to devitrification, in the temperature range in which several days' heating is required to obtain maximal density (see No. 131, Table 2b). In some cases the opalescence is so slight that it can be detected only by scattered light, when the pieces of glass are immersed in liquid of about the same refractive index. Sometimes it is not possible to separate the change in density resulting from such slight devitrification from the change in glass density which is under consideration; although more frequently a different order of magnitude in the rate of change in density makes it possible to avoid confusion.

Several of the harder glasses were heated to give maximal density in a few hours at 500–550°, and then heated for about 4 months at 230°, or 3 months at 330°. They did not change. Samples of the same or similar glasses which were low in density increased in density, but did not quite reach the maximal value.

Glasses that reach a maximal density in a few hours at 500–550° may require more than a week to reach a higher maximum at 400°.

The observed total density differences have not exceeded 0.018, except for No. 68a, in which the difference was 0.024, and for glasses highest in lime the change has been about 0.008 and still less for those very high in silica. Most of the change has been an increase over the original density.[14]

The annealing procedure adopted fulfilled our object of making measurements on glasses in a reproducible condition, but whether or not the glasses are exactly comparable is an open question. We have no generally accepted explanation of what takes place in glass during annealing, in addition to the removal of mechanical strain, and not until

[14] Differences of 0.004 in density of different parts of No. 9a, which was very rapidly cooled, disappeared on annealing. Usually differences as large as this could be related to chemical inhomogeneity.

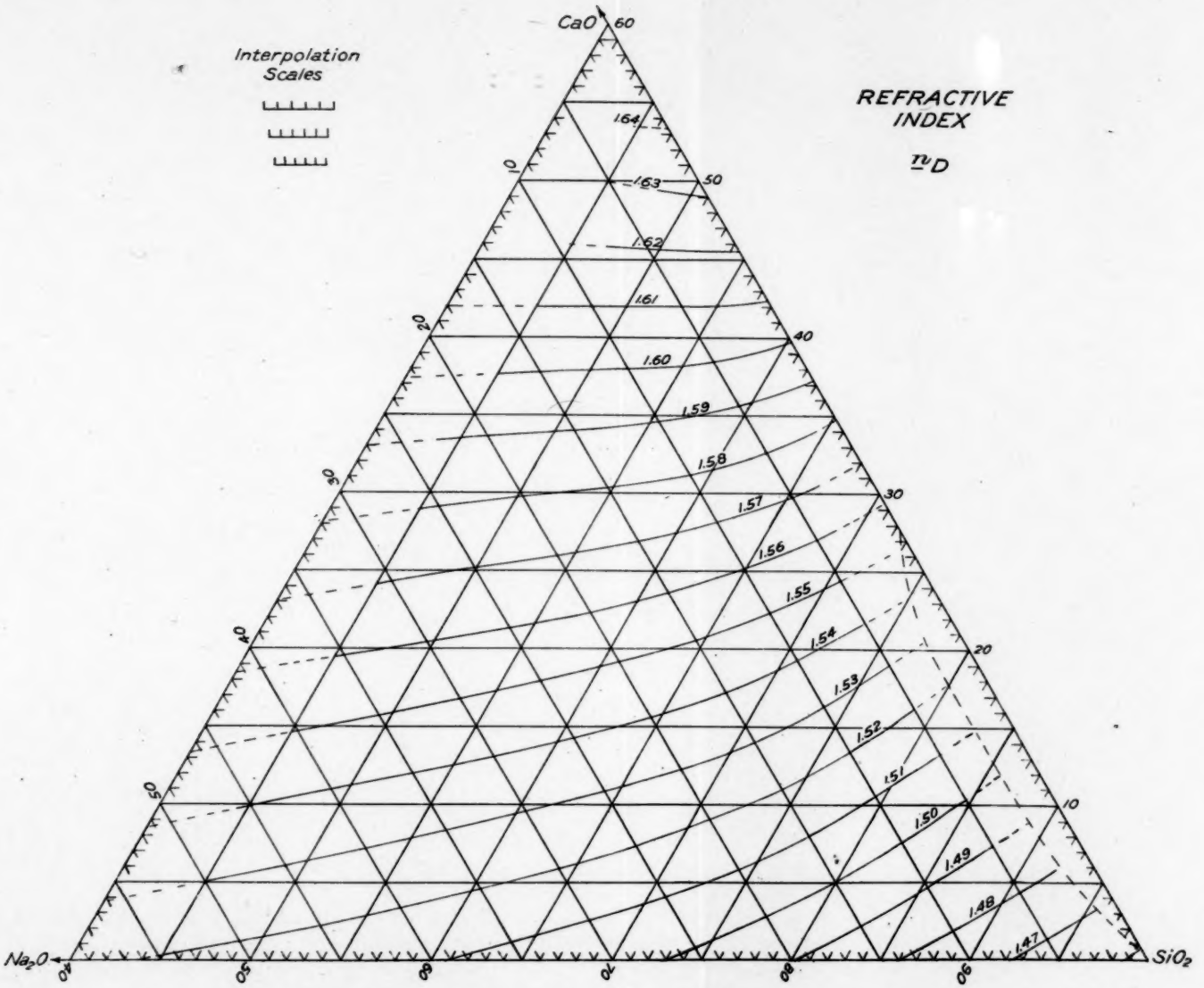


FIG. 3. *The relation between density and composition of glasses in the ternary system, $Na_2O-CaO-SiO_2$.*

we have some knowledge of the processes taking place can we select a heat treatment which will give us glasses in a corresponding state, if such a term has any meaning. It might be argued that it would be better to anneal each glass at its "annealing temperature," by which is meant an experimentally obtained temperature dependent upon composition, dimensions, time and rate of cooling, to which commercial glasses are subjected in order to bring about desirable changes. The temperature so determined is an arbitrarily selected point of the viscosity-temperature curve, and there is no reason to assume that the process whose progress is measured by the change in density or refractive index would come to a definite conclusion at a definite viscosity. In our opinion, the heat treatment we have selected, designed to bring out a maximal density or refractive index, gives more nearly comparable data than would have been obtained at the "annealing temperature."

THE DENSITY AND OPTICAL PROPERTIES OF THE GLASSES

The final results on the density and optical properties of the glasses are assembled in Tables 3 and 4, and presented graphically in Figs. 3 to 7. In Table 3 the glasses are arranged in order of decreasing SiO_2 content, and the compositions, determined as already described, are given in the second, third, and fourth columns; values enclosed in parentheses were obtained by difference. These compositions are plotted in Fig. 1. Column 5 gives the observed density at 20° of the annealed glasses with reference to water at 4°, while column 6 gives the corrections to add to the observed values to fit the smoothed curves of Fig. 3. In most cases determinations were made on both unannealed and annealed glasses, and column 7 gives the experimentally determined increase in density on annealing. Some glasses were not successfully annealed, usually because of devitrification, and in such cases an estimated difference, indicated by parentheses in column 7, was added to the density of the unannealed glass. Columns 8, 9, and 10 give, respectively, the observed refractive index, n_D, on annealed glasses, the correction to be added to fit the smoothed curves of Fig. 4, and the change in index on annealing, with the values in parentheses having a significance similar to those in column 7. Columns 11 and 12 give the observed values of dispersion $(F-C)$, and the corrections to be added to fit the smoothed curves of Fig. 5.

The probable error in determining the density is ± 0.001, and in n_D, ± 0.0005. The latter takes account: (1) of widening of the signal by inhomogeneities; and (2) of an error of $1'$ in measuring the prism. The

TABLE 3.

No.	Na_2O	CaO	SiO_2	Observed $d^{25}/_4$ annealed	×10⁻⁵ Add to observed d to obtain d of curves	×10⁻⁵ d increased by annealing	Observed n_D annealed	×10⁻³ Add to observed n to obtain n of curves	×10⁻⁴ n increased by annealing	×10⁻⁴ Observed $F-C$	×10⁻⁴ Add to observed $(F-C)$ to obtain $(F-C)$ of curves
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	—	—	99.90	2.203	0	0	1.458$_5$	0	0	67.$_5$	0
1	5.00	—	95.00	2.240	+ 1	2	1.465$_5$	0	9	72	0
1a	10.00	—	90.00	2.291	0	5	1.474	− ½	10	75.$_1$	− ½
2	3.90	5.70	(90.40)	2.310	+ 1	(5)	1.483	− ½	(10)	—	—
2a	6.00	4.00	90.00	2.311	− 2	4	1.480$_5$	0	6	75.$_5$	− ½
3	11.83	—	(88.17)	—	—	—	1.476$_5$	0	8	75.$_1$	+ ½
3x	13.87	—	86.13	2.329	0	9	—	—	—	—	—
3a	10.89	3.01	86.10	2.345	− 1	3	1.486	0	12	78	0
3b	8.00	6.00	86.00	2.355	+ 2	5	1.490$_5$	0	8	78	0
4	14.86	—	(85.14)	2.334	+ 4	2	1.481	0	0	77.$_5$	1
5	9.21	7.58	83.21	2.389	+ 4	7	1.498$_5$	0	14	80.$_5$	+ 1
5a	17.95	—	82.05	2.369	0	9	—	—	—	—	—
6	6.55	11.65	81.80	2.421	+ 1	6	1.507	+ ½	—	—	—
7	19.55	—	(80.45)	2.383	+ 1	8	1.490	− ½	10	82.$_5$	− ½
8	16.98	5.01	78.01	2.432	0	6	1.503$_5$	0	—	85	−1
9	12.0	10.0	78.0	2.455	+ 1	6	1.511	+ ½	—	85	0
9a	22.54	—	77.46	2.414	− 1	9	—	—	—	—	—
10	14.93	8.01	77.06	2.456	0	6	1.510	0	—	85.$_5$	0
11	12.85	11.03	76.12	2.485	− 3	7	1.517	0	11	87	− ½
12	8.90	14.99	76.11	2.497	+ 1	5	1.523	0	6	87	0
13	18.89	5.01	76.10	2.455	− 5	11	1.506	0	16	85.$_5$	+ ½
14	6.18	17.96	75.86	2.505	+ 4	6	1.527$_5$	+1	—	88	− ½
15	18.64	5.76	75.60	2.468	−10	9	1.509	−1	15	—	
16	4.48	20.23	75.29	2.535	− 7	—	1.533	+ ½	—	—	
17	17.25	7.54	75.21	2.475	− 1	8	1.512	− ½	—	88	−1
18	24.81	—	(75.19)	2.429	+ 1	10	1.497$_5$	0	15	87	−1
19	25.00	—	(75.00)	2.431	0	10	1.497$_5$	0	12	86.$_5$	− ½
20	17.50	7.50	75.00	2.477	− 1	5	1.512$_5$	− ½	—	87	0
21	15.04	9.98	74.98	2.489	− 1	—	1.517	0	10	87.$_5$	0
22	(13.40)	11.64	74.96	2.495	+ 2	—	1.519	+ ½	—	88	− ½
23	10.08	14.97	(74.95)	2.506	+ 5	—	1.525	+ ½	—	87	+ ½
24	15.20	10.10	74.70	2.489	+ 2	14	1.517	+ ½	15	87	+ ½
25	15.28	10.03	74.69	2.492	− 1	—	1.517$_5$	− ½	—	86.$_5$	+1
26	(5.37)	20.25	74.38	2.532	+ 6	(10)	—	—	—	—	—
27	(5.46)	20.25	74.29	2.536	+ 3	14	1.534$_5$	+1	—	88.$_5$	+ ½
28	5.73	20.02	(74.25)	2.522	+16	10	1.532$_5$	+2½	4	89	0
29	12.82	12.98	74.20	2.509	+ 1	8	1.524	− ½	—	88.$_5$	− ½
30	18.48	7.34	74.18	2.484	− 6	9	1.513$_5$	− ½	12	88$_5$	−1
31	20.73	5.12	74.15	2.469	− 2	16	1.508$_5$	+ ½	15	87.$_5$	0
33	13.88	12.02	74.10	2.505	+ 1	7	1.522	0	—	88.$_5$	− ½
34	15.04	11.03	73.93	2.505	− 2	—	1.521	− ½		89	−1
36	7.32	19.04	73.64	2.541	0	5	1.535	0	—	90	− ½
37	12.95	13.44	73.61	2.518	− 1	—	1.526	− ½	—	88	+ ½
38	13.30	13.13	73.57	2.519	− 3	—	1.526	− 1		89.$_5$	−1
39	16.42	10.02	73.56	2.501	0		1.519$_5$	− ½		88	0
40	(23.48)	3.07	73.45	2.461	0	13	1.505	+ ½	11	87	+ ½
41	21.71	5.00	73.29	2.474	− 1	6	1.509$_5$	0		88	0
42	13.81	13.02	73.17	2.519	0	7	1.525$_5$	− ½		88	+1
42a	22.84	4.01	73.15	2.470	− 2	8	1.508	− ½		87.$_5$	0
43	17.91	9.01	73.08	2.500	− 1	—	1.518	− ½	—	88.$_5$	0
44	27.32	—	(72.68)	2.449	0	11	1.500$_5$	0	—	89	−1½

TABLE 3. (*Continued*).

No.	Na_2O	CaO	SiO_2	Observed $d^{20}/_4$ annealed	×10⁻⁶ Add to observed d to obtain d of curves	×10⁻⁶ d increased by annealing	Observed n_D annealed	×10⁻⁵ Add to observed n to obtain n of curves	×10⁻⁴ n increased by annealing	×10⁻⁴ Observed $F-C$	×10⁻⁴ Add to observed $(F-C)$ to obtain $(F-C)$ of curves
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
45	11.82	16.03	72.15	2.546	− 2	9	1.533	− ½	—	90.₅	− ½
46	8.92	19.02	72.06	2.560	− 2	—	1.538₅	−1	—	92	−1
47	(21.66)	6.54	71.80	2.495	− 2	16	1.514	0	11	89	0
48	18.52	10.10	71.38	2.518	− 1	7	1.522	− ½	12	91	−1
49	(21.35)	7.54	71.11	2.499	+ 5	8	1.515₅	+1½	15	90	0
50	10.91	18.02	71.07	2.561	+ 2	5	1.537₅	0	—	92	− ½
50a	5.46	23.66	70.88	2.594	0	11	1.549	−1	10	93	− ½
51	29.20	—	(70.80)	2.459	+ 2	10	1.502	0	10	—	—
52	9.11	20.19	70.70	2.585	− 8		1.544	−2		93.₅	−1½
53	24.02	5.32	70.66	2.495	− 1	6	1.513	0	—	90	0
54	(13.78)	15.66	70.56	2.553	+ 2	10	1.533₅	0	15	91	+ ½
55	15.68	14.04	70.28	2.549	− 1	9	1.531₅	− ½	—	92	− ½
56	14.77	15.00	70.23	2.557	− 3	—	1.533₅	− ½		92.₅	−1
57	23.26	6.52	70.22	2.504	0	5	1.516	− ½		91	0
58	24.86	4.94	70.20	2.499	− 4	10	1.513	− ½	10	91	− ½
59	15.91	14.08	70.01	2.548	+ 3	7	1.531	+ ½	—	92	− ½
60	24.71	5.31	69.98	2.496	+ 3	3	1.513₅	0	10	91.₅	0
60a	(25.55)	4.61	69.84	2.503	− 8	14	1.514	−2	(15)	90.₅	+1
61	(17.23)	13.00	69.77	2.540	+ 6	12	1.528₅	+1	15	90.₅	+1
62	25.79	5.01	69.20	2.504	− 2	7	1.513₅	0	—	91.₅	0
63	5.56	25.40	(69.04)	2.609	+11	12	1.552₅	+2½	6	94	0
64	20.54	10.78	68.68	2.544	− 5	9	1.526₅	− ½	—	93	−1
65	20.90	10.84	68.26	2.543	0	7	1.527	− ½	—	92.₅	0
66	13.83	18.04	68.13	2.586	+ 2	6	1.541₅	0	—	94	− ½
67	24.95	7.00	68.05	2.520	0	5	1.518₅	+ ½		92.₅	0
68	22.10	10.14	(67.76)	2.543	− 2	—	1.525₅	0		—	—
68a	32.43	—	67.57	2.484	− 2	24					
69	19.64	13.03	67.33	2.561	+ 1	—	1.532	0		93	+ ½
70	(3.87)	28.84	67.29	2.655	+ 1	5	1.564₅	0		97	− ½
71	17.45	15.57	66.98	2.582	− 3	—	1.538	0		94	0
72	5.98	27.24	66.78	2.654	0	5	1.563	− ½	6	97	0
73	(12.44)	20.79	66.77	2.613	0	10	1.549	− ½	5	94.₅	− ½
74	23.45	10.07	66.48	2.547	+ 1	5	1.526₅	0		94	− ½
75	(32.20)	1.34	66.46	—	—	—	1.506₅	+1½	(10)	91	+1½
76	18.69	14.91	66.40	2.581	− 2	—	1.537	0		94.₅	0
77	33.86	—	66.14	2.491	0	6	1.506	0	(5)	—	
78	17.80	16.04	66.16	2.591	− 2	—	1.540	0		94.₅	0
79	18.95	15.06	65.99	2.584	0	—	1.538	0		—	
81	19.15	15.00	65.85	2.586	− 3	10	1.538	0		—	
82	9.07	25.28	65.65	2.656	− 0	16	1.559	+ ½	15	97	0
83	16.64	17.83	(65.53)	2.595	+11	—	1.542₅	+1	—	95.₅	0
84	21.80	13.09	(65.11)	2.573	+ 3	—	1.533₅	+ ½		95	0
85	10.53	24.38	(65.09)	2.649	0	5	1.559	− ½	—	97.₅	− ½
86	30.87	4.27	(64.86)	—	—	—	1.515	+1		94	0
87	32.82	2.40	64.78	2.509	+ 1	(7)	1.511	+1	(10)	93.₅	+ ½
87a	35.25	—	(64.75)	2.498	0	(8)	1.507₅	− ½	(10)	94	− ½
88	32.43	3.02	64.55	2.517	− 2	(7)	1.513	0	(10)	94	0
88a	35.50	—	(64.50)	2.500	− 1	9	1.507	0	(10)	92.₅	+1
89	27.20	8.33	(64.47)	2.546	+ 2	—	1.524	+ ½	—	96	− ½
90	14.15	21.47	64.38	2.638	− 4	8	1.553	− ½	—	97	0
92	(8.22)	28.51	63.27	2.691	− 4	7	1.569₅	0	—	100	− ½

TABLE 3. (*Continued*).

No.	Na_2O	CaO	SiO_2	Observed $d^{20°/4°}$ annealed	$\times 10^{-3}$ Add to observed d to obtain d of curves	$\times 10^{-3}$ d increased by annealing	Observed n_D annealed	$\times 10^{-3}$ Add to observed n to obtain n of curves	$\times 10^{-4}$ n increased by annealing	$\times 10^{-4}$ Observed $F-C$	$\times 10^{-4}$ Add to observed $(F-C)$ to obtain $(F-C)$ of curves
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
93	—	37.34	62.50	2.746	+ 3	12	1.590$_5$	0	5	103	0
94	5.11	32.59	62.30	2.719	+ 2	4+	1.580$_5$	0	12	—	—
95	9.71	28.18	62.11	2.694	0	6	1.570$_5$	0	6	100.$_5$	0
96	38.51	—	61.49	2.515	0	—	1.509$_5$	0	—	95.$_5$	+ ½
96a	28.60	10.04	61.36	2.575	+ 1	3	1.529$_5$	+ ½	1	97	+1
97	10.42	28.46	61.12	2.702	0	5+	1.571	+ ½	13	100.$_5$	+ ½
98	7.44	32.17	(60.39)	2.729	+ 1	10	1.581$_5$	− ½	12	102.$_5$	0
98a	39.66	—	60.34	2.521	0	16	1.511	—	1	—	—
99	16.81	23.04	60.15	2.672	− 2	9	1.560	0	—	100	+ ½
100	19.99	20.00	60.01	2.657	− 7	8	1.553$_5$	− ½	—	100	0
101	5.01	35.70	59.29	2.761	+ 2	7	1.591	− ½	6	105	− ½
101a	34.08	6.66	59.26	2.562	+ 2	2	1.523$_5$	+1	3	100	−1
102	9.97	31.14	58.89	2.733	− 1	5	1.580	0	3	104.$_5$	− ½
103	26.22	15.00	58.78	2.618	+ 3	6	1.542$_5$	+ ½	—	100	+ ½
104	17.14	24.24	58.62	2.692	− 6	15	1.563	+ ½	—	102	0
105	28.47	14.31	57.22	2.626	− 2	5	1.542$_5$	0		101	+ ½
106	15.79	27.07	57.14	2.712	0	—	1.571	0	—	104	0
107	—	42.89	(57.11)	2.835	0	9	1.612	0		102.$_5$	− ½
107a	33.91	9.04	57.05	2.588	+ 2	1	1.531	0	4	101	0
107b	13.80	29.40	(56.80)	2.735	− 6	—	—	—		—	—
108	23.06	20.15	56.79	2.665	+ 2	8	1.555	+ ½	17	102.$_5$	0
109	44.32	—	(55.68)	2.546	− 5	(5)	1.514	− ½	(10)	100.$_5$	½
110	23.50	21.14	(55.36)	2.683	− 4		1.559$_5$	− ½	10	105.$_5$	−1½
111	9.97	34.72	55.31	2.773	0	4	1.590	+ ½	4	108.$_5$	−1
112	16.41	28.31	(55.28)	2.733	− 5	7	1.577	−2	10	106.$_5$	−1
113	33.87	10.87	(55.26)	2.610	0	—	1.536$_5$	0	—	102	+1
114	6.13	38.75	55.12	2.808	− 4	6	1.602$_5$	−1	11	109	− ½
115	4.56	40.42	(55.02)	2.816	+ 2	—	1.606$_5$	− ½	—	110.$_5$	−1
116	41.94	3.33	(54.73)	2.568	− 3	≥	1.521$_5$	− ½	—	101.$_5$	+1
117	38.32	7.50	54.48	2.587	+ 5	—	1.529	+ ½	—	101.$_5$	+2
118	31.33	14.71	(53.96)	2.630	+11	—	1.543$_5$	+1½	—	102.$_5$	+1½
119	21.06	24.97	53.97	2.705	+ 5	6	1.567	+1	13	104.$_5$	+1½
120	4.44	42.04	53.52	2.838	− 2	6	1.610$_5$	− ½	10	111	0
121	5.18	41.50	(53.32)	2.832	− 1	6	1.610	−1	5	109.$_5$	+2
122	46.69	—	(53.31)	—	—		1.515	0	(10)	103.$_5$	− ½
122a	28.44	18.29	53.27	2.664	+ 3	2	1.552$_5$	+ ½	0	107	−1½
122b	39.91	7.52	52.57	2.601	0	5	1.531	0	11	105	0
122c	20.80	27.10	(52.10)	2.730	0	—	1.574$_5$		—	108	− ½
123	31.87	16.08	(52.05)	2.665	− 7	14	1.550$_5$	−1½	15	106	0
124	10.23	37.98	51.79	2.807	+ 1	3	1.599$_5$	+ ½	3	110.$_5$	+ ½
125	—	48.26	(51.74)	2.898		6	1.629$_5$	−1	0	115.$_5$	−1½
126	10.61	37.63	(51.76)	2.802	+ 4		1.598$_5$	+ ½	—	111	0
126a	(23.49)	24.95	51.56	2.712	+ 6	6	1.567$_5$	+1	(12)	—	—
127	18.00	30.65	51.35	2.761	− 4	13	1.584	−2½	15	108	+1
128	39.63	9.16	51.21				1.538	−3		—	—
128a	14.68	34.13	51.19	2.777	+ 4	7	1.589$_5$	+1	7	109.$_5$	+ ½
129	9.88	39.03	(51.09)	2.821	− 3	7	1.603	− ½	8	111.$_5$	0
130	17.38	31.65	50.97	2.764	0	9	1.584	0	—	109	+1
131	36.37	12.66	50.97	2.648	− 8	13	1.543$_5$	−1	13	107	− ½
131a	23.99	25.10	50.91	2.719	+ 2	7	1.568$_5$	+1	8	109	−1
131b	(37.44)	11.68	50.88	2.641	− 8	(10)	1.540$_5$	0	(10)	105.$_5$	+1

TABLE 3. (*Continued*).

No.	Na_2O	CaO	SiO_2	Observed $d^{20/4}$ annealed	$\times 10^{-3}$ Add to observed d to obtain d of curves	$\times 10^{-3}$ d increased by annealing	Observed n_D annealed	$\times 10^{-3}$ Add to observed n to obtain n of curves	$\times 10^{-4}$ n increased by annealing	$\times 10^{-4}$ Observed $F-C$	$\times 10^{-4}$ Add to observed $(F-C)$ to obtain $-(F-C)$ of curves
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
131c	17.52	31.61	50.87	2.761	+ 3	8	—	—	—	—	—
132	49.20	—	(50.80)	2.563	0	(8)	1.517	− ½	(10)	105	0
132a	9.60	39.64	50.76	2.820	+ 2	2	1.604	0	—	112	0
132b	21.40	27.90	50.70	2.741	0	—	1.576	− ½	—	—	—
133	(33.47)	15.98	50.55	2.670	− 8	18	1.550	− ½	(15)	107	0
134	29.77	20.18	50.05	2.696	−16	20	1.560₅	−1½	10	109	−1
134a	34.42	15.54	50.04	2.663	− 2	5	1.549₅	− ½	11	108	− ½
135	26.81	23.57	(49.62)	2.710	+ 5	(9)	1.566	+1	(10)	109	0
136	5.30	45.44	49.26	2.867	+ 3	—	1.620₅	−1	—	113.₅	+1
136a	50.78	—	49.22								
137	(28.42)	22.46	49.12	2.717	− 7	15	—	—	—	109.₅	0
137a	30.00	21.00	49.00	2.697	+ 3	8	1.561	+ ½	20	—	—
138	(7.40)	44.30	48.30	2.859	+ 3	—	1.616₅	− ½	—	114.₅	+ ½
139	—	51.20	(48.80)	2.918	0	8	1.635	0	2	116	+1
140	52.06	—	47.94	2.575	− 2	(8)	1.518	0	(10)	—	—
141	16.98	35.54	(47.48)	2.794	+ 6	—	1.594₅	0	—	112.₅	+ ½
142	40.76	12.32	(46.92)	2.654	− 1	(8)	1.544₅	− ½	10	109.₅	+ ½
143	7.98	45.16	46.86	2.863	+ 6	(8)	1.616₅	+2	(5)	115	+1½
144	36.50	17.27	46.23	2.681	+ 2	7	1.555	0	8	110	+1
145	44.77	9.05	46.18	2.630	+ 5	—	1.539	−1	10	111	− ½
146	44.56	10.07	45.37	2.640	+ 4	3	1.540₅	0	2	111.₅	+ ½
147	9.27	45.38	45.35	2.874	− 1	8	1.619	0	9	118	− ½
148	14.47	40.24	45.29	2.837	0	4	1.606₅	− ½	12	113	−1½
149	4.79	50.06	45.15	2.902	+ 6	(7)	1.630₅	0	(10)	120	−1
150	34.91	20.02	45.07	2.703	+ 2	5	1.562	−1	7	111.₅	+ ½
150a	55.00	—	45.00	—	—	—	1.519	+ ½	7		
151	22.60	32.50	44.90	2.784	+ 2	13	1.587₅	+ ½	(10)	114.₅	0
152	—	55.83	44.17	2.953	0	6	1.645₅	0	5	121.₅	0
153	29.00	27.18	43.82	—			1.578₅	−1½	(7)	114.₅	− ½
154	20.88	38.16	40.96	—			1.599	0	(5)	119	0
155	26.86	32.24	40.90	2.800	− 7	13	1.587	+1	(7)	—	—
156	38.91	23.43	37.66	2.742	0	(7)	1.574₅	−2	(5)	—	—

Numbers refer to the like numbered glasses.

0. A clear bead made from quartz in an oxy-hydrogen flame had a density of 2.203. The clearest pieces of Day and Shepherd's glass (Sci., *23*, p. 670; 1906) had d 2.204. A disk of optical silica had d 2.206. Annealing these either at 700° or at 1000° increased d not more than 0.001. For Day and Shepherd's glass n_D 1.458₈. $(F-C)$ is from the literature. Cf. "The Properties of Silica," R. B. Sosman.

1. Almost free of bubbles and striae. Fined at 1650°, quenched in mercury.

1a, 2. Same; fined at 1620°, quenched in mercury.

3a. Same; fined at 1550°, quenched in mercury.

5. Milky after 15 minutes at 650°.

9. Last fining 1 hr. at 1500°, loss 0.03 percent.

10. Fined 16 hr. at 1350°; lost 0.01 percent.

13. Became opalescent and corners rounded at 600° for several hours, then annealed at 460°. Duplicate tests of changes in n and d agreed.

14. Liquidus 1438°; fined at 1475°; quenched in H_2O.

15. Clear glass after annealing but slightly striated; large piece used. d redetermined.

16, 26, 27, 28. These four glasses were all attempts to make a glass of approximately 5 percent Na_2O, 20 percent CaO, 75 percent SiO_2. Liquidus temperature 1475°; hard to get homogeneous, and very difficult to quench, even in mercury, without devitrification. Final quenching in small lots in mercury. Distinctly opalescent when first prepared, not distinctly more so when annealed.

31. The first glass melted in this study; $Fe_2O_3+Al_2O_3=0.12$ percent.

52. Fined at 1550°.

60. Distinctly opalescent after annealing, barely so before. Changes due to annealing accurately determined in duplicate.

63. Difficult to quench, even in mercury; opalescent, striated.

82. Final melting, 1500°; quenched in H_2O.

93. Difficult to quench, even in mercury.

97. Composition of the compound, $Na_2O \cdot 3CaO \cdot 6SiO_2$.

101. Fined at 1500°; liquidus 1472°; quenched in mercury.

108. Very slightly opalescent when annealed.

114. Liquidus 1493°; melted 1550°; quenched in H_2O.

119. Only most rapid quenching in thimble crucible gives clear glass which anneals.

122a. Trace of opalescence both before and after annealing.

125. Slightly milky after 15 min. at 650, but not after 3 hrs. at 565°.

127. Milky after 15 min. at 650°, but not after 3 hrs. at 565°. Very close to composition of compound, $Na_2O \cdot 2CaO \cdot 3SiO_2$.

130. Composition of compound, $Na_2O \cdot 2CaO \cdot 3SiO_2$.

131a. Milky after 6 hrs. at 475°; clear after 4 hrs. at 466°.

133. Quenched in H_2O.

134. Slightly opalescent before and more so after annealing; changes partly due to crystallization.

134a. Practically composition of compound, $2Na_2O \cdot CaO \cdot 3SiO_2$. Necessary to quench in H_2O. Barely trace of opalescence after 4 hrs. at 466, same d after 1½ hrs. at 466°.

135. Clear glass; slightly milky when annealed 12 hrs. at 495°, but n increased 0.020 and d increased 0.031. Microscope shows many minute equant apparently isotropic grains of higher n than glass.

137. Trace opalescence after annealing.

137a. Very milky in 15 hrs. at 475°, $d=2.730$, $n=1.566$; trace of opalescence in 2 hrs. at 475°.

144. Quenched in H_2O.

146. Annealed 2 hrs. at 460°; did not devitrify.

147. Quenched in mercury.

148. Quenched in mercury.

149. Has to be quenched in mercury in foil by dropping; some grains obtained 2 mm thick.

150. Quenched in mercury.

153. Foil. Drop quench into mercury 2 mm thick considerably crystallized except at surface ∴ no d, but n on edge of prism was obtained. Attempts to anneal caused crystallization.

154. Liquidus 1570°; foil; drop quench in mercury.

155. Liquidus 1570°; foil; drop quench in mercury.

observed changes of density caused by annealing are probably accurate within ±0.002, and those of refractive index, within ±0.0004. However, in glasses most difficult to prepare or anneal the changes observed may have been slightly affected by devitrification. In some such glasses

n_D was measured on one prism before annealing, and on another after annealing.

The argument in all of the figures is weight percent of the three oxides. The use of molecular percent would not in this case significantly straighten the curves, and in any case linear relationships would not be affected by interchanging the methods of plotting.

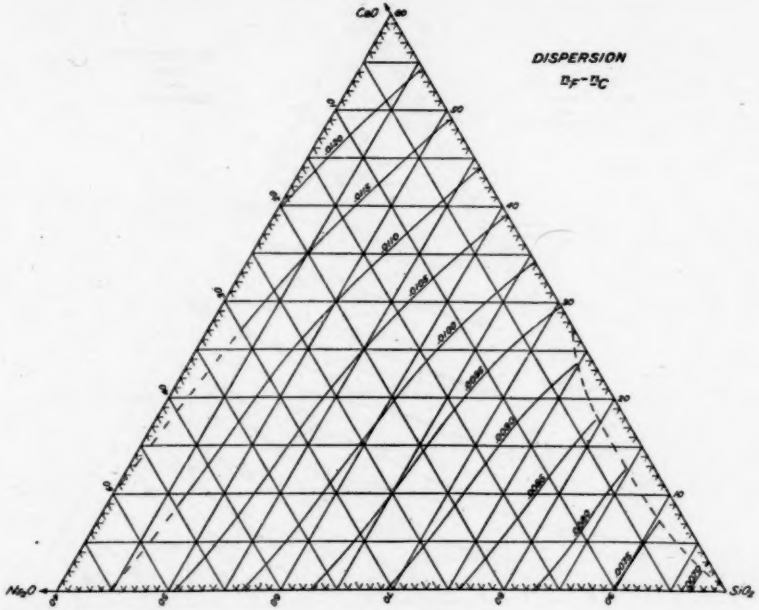


FIG. 5. *The relation between dispersion, $n_F - n_C$, and composition of glasses in the ternary system, $Na_2O - CaO - SiO_2$.*

Figs. 3 and 4 show the relation between the composition and the density and refractive index, respectively. Comparison between these two families of curves is facilitated by Fig. 6, in which both sets of curves are drawn. Some of the curves of constant property trend, over a portion of their courses, directly toward the soda apex, hence there is an area in each diagram in which the content of Na_2O is practically without effect on the property represented. For refractive index, the area is a narrow crescent, extending downward from *40* on the right to *50* on the left margin. For density the crescent extends downward from *41* on the right to *55* on the left. The two areas do not

overlap. CaO dominates both the density and the refractive index diagrams.

Fig. 5 shows the relation between dispersion, $(F-C)$, and composition, and a comparison between n_D and dispersion is shown in Fig. 7. In contrast to the density and refractive index, the dispersion is but slightly more affected by CaO than by Na_2O, and the lines of equal dis-

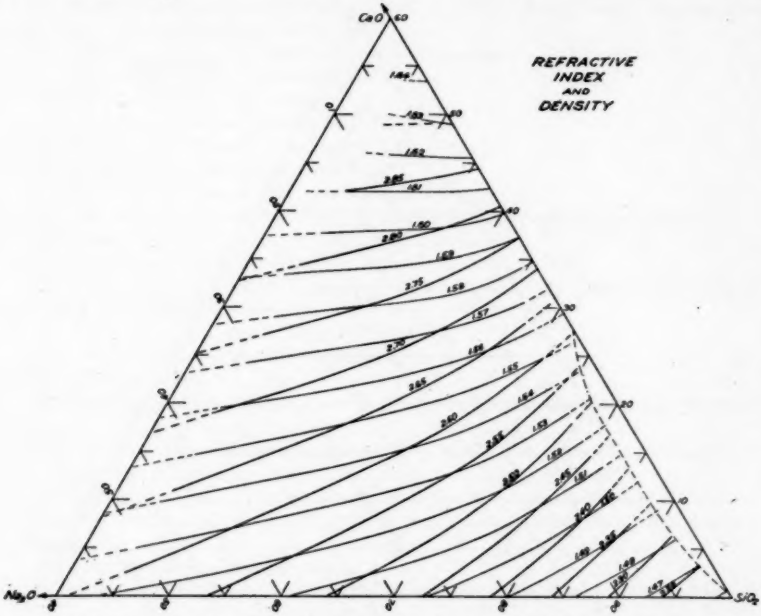


FIG. 6. *Comparison between the density and the refractive index of glasses in the ternary system, $Na_2O-CaO-SiO_2$.*

persion have a marked *S*-shape, and cross the diagram at but a slight angle to the lines of constant SiO_2 content. Neither Na_2O nor CaO produces large dispersion, therefore not great, although definite, differences were found in the shapes of the dispersion curves of their silicate glasses. Table 4 gives measurements on several glasses high in either Na_2O or CaO for a wide range of wave lengths, which measurements have been reduced to partial dispersions in both directions from *F*. The nearly equal dispersions, $h-F$ and $F-C$, toward violet and red respectively, may be compared. Silica glass is in the middle of the table, lime increases upward and soda downward. As dispersion increases by addition

of either oxide, the violet end of the spectrum becomes relatively longer, but the lengthening is about 5 percent more rapid for soda.[15] Calcium and barium silicate glasses are alike in this respect, as are soda and lead.

The derived quantity $\gamma = (n_D - 1)/(n_F - n_c)$ is used in lens design, and is usually given in optical glass tables. We have calculated the γ-value for our glasses, and Fig. 8 shows the lines of constant property plotted in the composition triangle.

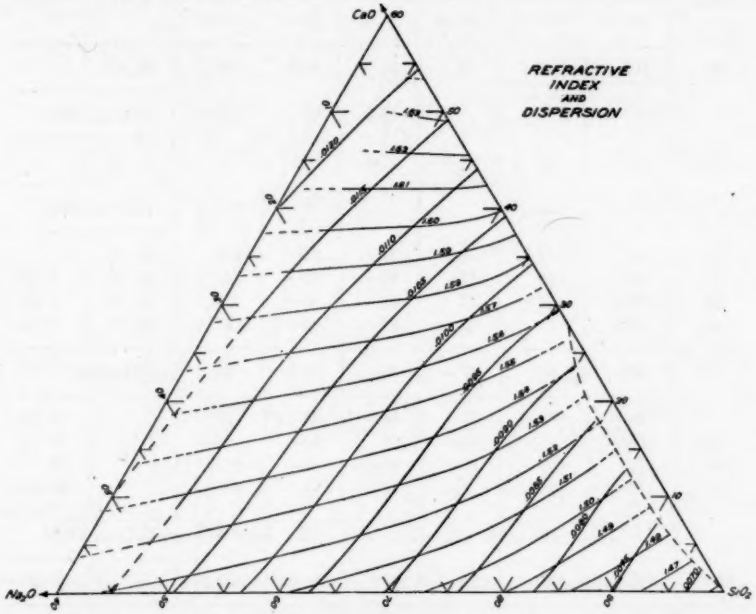


FIG. 7. *Comparison between the refractive index and dispersion of glasses in the ternary system,* $Na_2O-CaO-SiO_2$.

The diagrams were constructed from the experimental results by first plotting the measured values of the property according to composition, and sketching in trial iso-curves (curves of constant property). Then several straight lines or cross sections were drawn across the diagram nearly normal to the iso-curves, and to these lines were transferred the adjacent experimental values by movements parallel to the trial curves.

[15] On a chart like the one in Int. Critical Tables 7, 19, except that silica glass and the highest lime glass are drawn horizontal and corresponding points connected, the soda glasses slope slightly upward toward the violet.

From the change in property along the cross sections the trial iso-curves were more definitely located, but they were given a final correction by interpolations from all points adjacent to them, giving, however, more weight to the more perfect glasses.[16]

TABLE 4. *Dispersions of Glasses*

No.	×10⁻⁴						CaO	Na_2O
	F−b	*F−C*	*F−d*	*F−e*	*g−F*	*h−F*		
152	142	*121+*	85	56	67+	*124*	55.83	
		120+		55+	67	*123+*	BaO *ca.* 49%[a]	
139	136+	*117*	81+	53+	64	—	51.20	
		117				*119+*	BaO *ca.* 45%[b]	
125	135	*115*	79	52+	64	*116*	48.26	
121	129	*110*	77	50+	60+	*111*	41.50	5.18
98	120+	*102+*	72	47	56+	*103*	32.17	7.44
28	104+	*89*	62	41	48+	*88*	20.02	5.73
0	80	*68*	47+	30+	35+	*65*	SiO_2 glass	
3	88+	*75+*	52	34+	40+	*74*		11.83
88a	108+	*93*	64+	43	50+	*94+*		35.50
109	117	*100*	70+	46+	55+	*101+*		44.32
122	119+	*102+*	—	47+	57	*104+*		46.69
	—	*103*	—			*105+*	PbO *ca.* 13%[c]	
132	122+	*105*	73	48+	58+	*107+*		49.20

Wave lengths in μ: b=0.7065, C=0.6563, d=0.5876, e=0.5461, F=0.4861, g=0.4358, h=0.4047.

[a, b] Barium silicate optical glasses. Small amounts of other oxides do not affect the shapes of the dispersion curves in the fourth decimal place.

[c] A flint glass containing about 16 percent Na_2O. The 13 percent PbO changes dispersion about like 32 percent Na_2O.

These final interpolations, especially along the refractive index curves 1.52 to 1.55 and the density curves 2.50 to 2.65, include so many data that they might be expected to indicate whether or not the iso-curves

[16] In a few glasses the departures from the curves are relatively large and in the direction of faulty composition. These were among the first glasses made, of which the supply had been exhausted in the phase equilibrium work, so that the compositions could not be checked.

should be continuously curved or segmented into two or three less curved lines. No evidence of such segmentation was found. However, each might be replaced by three "best" straight lines without producing departures greater than the departures of single observations, but the distribution of departures would be too highly systematic to be tolerated.

Straight lines in these diagrams would necessarily imply additive relationships, in which case the straight segments of one line must be defi-

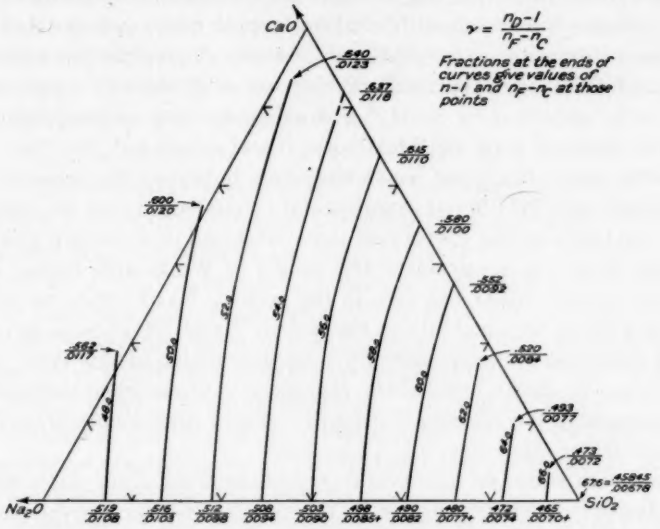


FIG. 8. *The relation between the quantity* $\nu = (n_D - 1)/(n_F - n_C)$ *and composition of glasses in the ternary system,* $Na_2O - CaO - SiO_2$.

nitely related to those of adjacent lines, and the only tenable assumption is that definite compounds determine the relationships. If such compounds are present between more than two of the end members of the triangle, or if more than one such ternary compound exists, then alternative constructions are possible, but experiment must decide as to which sets of three compounds determine the subsidiary triangles in each of which the additive relationship may hold. In those in which it does hold the iso-curves must be straight and parallel. But studies of solutions in general indicate that density, specific volume, refractive index, and dispersion are not additive. Thus, if the data should require the breaking up of each diagram such as Figs. 3, 4, and 5 into subsidiary

triangles, the iso-lines in them would not be straight and parallel. Nine triangles with apices at the eight compounds shown in Fig. 1 would need consideration, besides those that might result from hypothetical compounds.[17]

The argument of the preceding paragraph applies equally to binary systems, and we have tested our experimental results on density in the binary system, Na_2O-SiO_2, to ascertain if they are better represented by continuous curves or straight lines. The data may be represented by a number of intersecting straight lines, using either density or specific volume, but systematic deviations appear which indicate that continuous curves are to be preferred. Waterton[17] recently discussed the work of Winks and Turner,[18] making use of Masson's[19] equation, in the form employed by Scott,[20] and using, not the original data, but "those deduced from the ideal curve," and concluded that the compounds, $Na_2O \cdot 2SiO_2$ and $Na_2O \cdot 4SiO_2$, are indicated. We have applied the same equations to our experimental results, and found no justification for breaking the continuous curve obtained into straight line segments. Waterton recalculated the results of Winks and Turner from weight percent Na_2O and SiO_2 in the system, Na_2O-SiO_2, to weight percent $Na_2O \cdot SiO_2$ and SiO_2 in the system, $Na_2O \cdot SiO_2-SiO_2$, but such a recalculation is unnecessary for testing formulae of this type, for it easily can be shown[21] that if the expression is linear when composition is expressed in the one way it still will be linear when composition is expressed in the other way.

Inasmuch as one set of curves represents each set of our direct results satisfactorily, we may consider, on the same basis, some derived results.

[17] S. C. Waterton, J. Soc. Glass Tech., *15*, p. 354, p. 1931.
[18] F. Winks and W. E. S. Turner, J. Soc. Glass Tech., *15*, p. 185; 1931.
[19] D. O. Masson, Phil. Mag., *8*, 218; 1929.
[20] A. F. Scott, J. Phys. Chem., *35*, p. 2315; 1931.
[21] The formula used is

$$\frac{V_G-\frac{y'}{100}V_S}{x}=a_1\left(\frac{x}{V_G}\right)^{1/2}+b$$

in which V_S and V_G are the specific volumes of silica glass, and a glass containing x percent $Na_2O \cdot SiO_2$, y percent SiO_2. Let x', y' represent weight percent Na_2O and SiO_2, respectively. Then SiO_2 equivalent to $Na_2O=rx'$, in which r is the ratio of molecular weight of SiO_2 and Na_2O; $y=y'-rx'$; $x=x'(1+r)$. Substitution gives

$$\frac{V_G-\frac{y'}{100}V_S}{x}=a_1'\left(\frac{x'}{V_G}\right)^{1/2}+b'$$

in which a_1' and b' are the new constant coefficients $a_1(1+r)^{3/2}$ and $b(1+r)-rV_S$.

Specific refractivity has been calculated for the glasses by means of the following formulae: $(n-1)/d$, proposed by Gladstone and Dale;[22] $(n^2-1)/(n^2+2)\cdot 1/d$, proposed by Lorentz[23] and Lorenz;[24] $(n^2-1)/(n+0.4)\cdot 1/d$, proposed by Eykman;[25] and $(\log n)/d$, proposed by Lichtnecker.[26] For each equation the specific refractivity was calculated by assigning to each oxide an arbitrary constant value, and assuming that the specific refractivity is a linear function of the composition. Except for the Lorentz-Lorenz formula, the constant used for silica glass was that calculated from the best[27] values of refractive index and density, namely, $n_D = 1.4585$, $d = 2.203$. The other two constants were adjusted to give a fairly uniform distribution of positive and negative departures. The formulae for the Gladstone-Dale and the Lorentz-Lorenz equations were, respectively:

$$\text{Specific refractivity} = 0.1947N + 0.2268C + 0.2081S \quad (1)$$

$$\text{Specific refractivity} = 0.1115N + 0.1221C + 0.1237S^{28} \quad (2)$$

in which N, C, and S represent the percentages by weight, of Na_2O, CaO, and SiO_2. Considering the value of the coefficient of the weight percentage of SiO_2 as a constant, values of the refractive index and density of Na_2O and CaO which would satisfy these two expressions were obtained by treating them as simultaneous equations. By this means the values of 1.(586) and 2.(963) were obtained for the refractive index and density of Na_2O in the glassy state; 1.(777) and 3.(427) for the index and density of CaO in the glassy state. Nothing is known about the properties of crystalline Na_2O, but 1.83 and 3.37 are about the refractive index and density of crystalline CaO.[29]

The constants in the Eykman equation first were calculated from the rough values of the index and density of glassy Na_2O and CaO calculated from the Gladstone-Dale and Lorentz-Lorenz formulae, but the resulting departures were large and positive, and the constants required adjustment. The following formulae were used for calculating specific refractivity from the Eykman and Lichtnecker formulae, respectively:

$$\text{Specific refractivity} = 0.2530N + 0.2807C + 0.2753S \quad (3)$$

$$\text{Specific refractivity} = 0.6231N + 0.7508C + 0.7440S. \quad (4)$$

[22] I. H. Gladstone and T. P. Dale, Phil. Trans., p. 337; 1863.
[23] H. A. Lorentz, Wied Ann., *9*, p. 641; 1880.
[24] R. Lorenz, Wied Ann., *11*, p. 70; 1880.
[25] J. F. Eykman, Rec. trav. chim. Pays-Bas, *14*, p. 201; 1895.
[26] K. Lichtnecker, Phys. Zeits., *27*, pp. 116, 833; 1926.
[27] R. B. Sosman, *The Properties of Silica*, Chemical Catalog Co., New York, 1927.
[28] From the best values for silica glass, 0.1239+.
[29] Int. Crit. Tables, *1*, pp. 143, 165, 344.

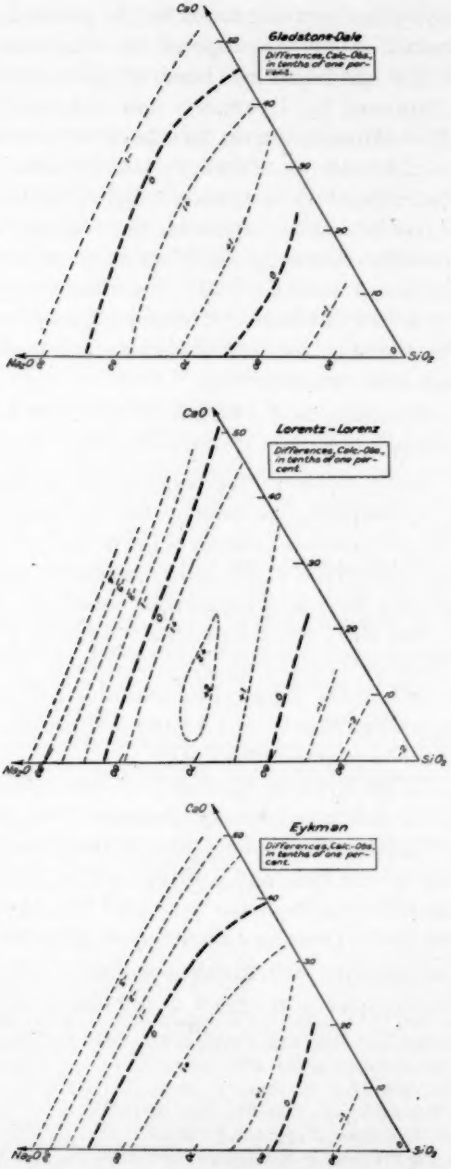


FIG. 9. *The percentage deviations from an additive relationship between composition and specific refractivity, calculated by means of: a, the Gladstone-Dale formula,* $(n-1)/d$; *b, the Lorentz-Lorenz formula,* $(n^2-1)/(n^2+2)$. $1/d$ *and, c, the Eykman formula* $(n^2-1)/(n+0.4)\cdot(1/d)$.

The percentage deviations from additivity for the first three formulae are plotted in Fig. 9, and the fourth formula gave a similar pattern of departures. The maximum contraction on mixing, of compositions within the diagram, is about 4 percent. While the general pattern of departures is the same for all the formulae tested, the Gladstone-Dale formula gives departures that are little more than one-half as large as the others plotted, and about one-third those of the Lichtnecker formula. For changes due to temperature in organic liquids, the Eykman formula is much superior to either the Gladstone-Dale or the Lorentz-Lorenz formula.[30]

DISCUSSION

Our results may now be compared with those of other recent investigators.[31] In order to do this it is necessary to consider the effect of the

[30] See J. F. Eykman, Rec. trav. chim. Pays-Bas, *14*, p. 201; 1895. We have made a random selection from later measurements of 10 liquids of various types with the following results, reduced to a volume change of 4 percent and expressed as percentage deviations: Gladstone-Dale, 0.2 percent; Lorentz-Lorenz, 0.35 percent; Eykman, 0.06 percent.

[31] Following is a brief summary of the earlier work on the relation between composition and properties of glass.

H. Hovestadt published a book (*Jena Glass and its Scientific and Industrial Applications*; English translation by J. D. and A. Everett, London, The MacMillan Co., 1902) dealing chiefly with the work done at Jena on various physical properties of glass; but the measurements were made on complex glasses whose compositions either are not given, or are calculated from the batch composition. While little direct information is given about optical properties, nevertheless until the last few years this was almost the only source of information on the properties of glass, and it remains the source of much information not obtainable elsewhere. E. Zschimmer, in Doelter's *Handbuch der Mineralchemie*, *7*, *1*, pp. 869–89, gives properties of a number of complex glasses, many of which are duplicates of those given by Hovestadt, and the same criticism as to uncertainty and complexity of composition apply to his data. F. E. Wright, in a book (*The Manufacture of Optical Glass and of Optical Systems*. Ordinance Dept. Document No. 2037. Government Printing Office, Washington, D. C.) includes a compilation of optical properties, density and composition, of the glasses mentioned by Hovestadt and by Zschimmer and in addition some chemical analyses of typical glasses. Much of the above information has been assembled by G. W. Morey in the International Critical Tables (International Critical Tables, *2*, pp. 87–106; 1927). The first attempt to establish a more accurate relation between density and refractive index and composition was that of E. S. Larsen (Am. J. Sci., *28*, p. 263; 1909), who worked with the albite ($Na_2O \cdot Al_2O_3 \cdot 6SiO_2$)-anorthite ($CaO \cdot Al_2O_3 \cdot 2SiO_2$) series, and also the $CaO \cdot SiO_2 - MgO \cdot SiO_2$ series of unannealed glasses. The results were discussed with the aid of the Gladstone-Dale and the Lorentz-Lorenz formulae for specific refractivity. Neither the refractive index nor the specific volume was found to be additive, but specific refraction is nearly additive. E. W. Tillotson (J. Ind. Eng. Chem., *3*, pp. 631, 897; 1911; ibid, *4*, pp. 246, 651, 820, 882; 1912; J. Am. Ceram. Soc., *1*, p. 76; 1918) prepared and made measurements on some glasses of the systems, $Na_2O - SiO_2$, $Na_2O - BaO - SiO_2$ and $Na_2O - CaO - SiO_2$. He concluded that his results were represented best on certain graphs by a series of straight lines intersecting at points assumed to represent hypothetical compounds. The work was not adequate to justify the conclusions, and was criticized to this effect by C. H. Kerr (J. Am. Ceram. Soc., *1*, p. 88; 1918). A voluminous tabulation of data has been published recently by W. Eitel, M. Pirani, and K. Scheel, *Glastechnische Tabellen*, Verlag Julius Springer, Berlin, 1932.

small amounts of impurities present in some of the glasses measured by others. In these particular glasses corrections may be made as follows:[32] Mg Omay be added to Na_2O; Fe_2O_3 may be added to CaO; Al_2O_3 may be omitted, and the analysis recalculated to 100 percent.

In Fig. 10 are plotted the density and refractive index, n_D, for the glasses in the binary system, Na_2O-SiO_2, as determined by Peddle,[33]

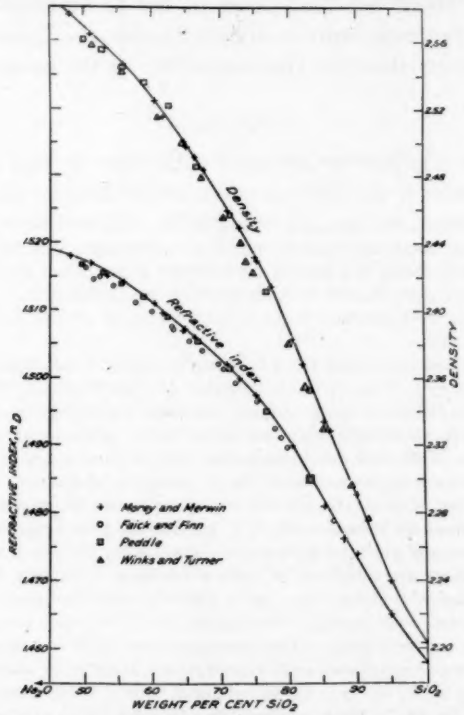


FIG. 10. *The refractive index and density of glasses in the binary system, Na_2O-SiO_2, as determined by several observers.*

by Faick and Finn,[34] by Winks and Turner,[35] and by us. The measurements of Peddle were made on glasses the compositions of which were calculated from the proportions of the ingredients melted, and which were melted in clay pots, but the results of Faick and Finn, of Winks

[32] Details to be presented in a later paper.
[33] C. J. Peddle, J. Soc. Glass Tech., *4*, p. 3; 1920.
[34] C. A. Faick and A. N. Finn, J. Am. Ceram. Soc., *12*, p. 753; 1931.
[35] F. Winks and W. E. S. Turner, J. Soc. Glass Tech., *15*, p. 185; 1931.

and Turner, and our own, all refer to glasses of known composition. Faick and Finn state that (p. 527) their purpose was "to anneal . . . at high temperatures . . . ," and their annealing temperatures ranged from 415° for the glass highest in Na_2O to 565° for that highest in SiO_2. Winks and Turner state only that their glasses were carefully annealed. But, judging from their previous work, their annealings must have been done at temperatures decidedly higher than ours. Our results are higher than the others, except a few of Peddle's, by amounts that can be accounted for by the lower temperatures and longer times of our annealings.

Peddle[36] made measurements on two series of soda-lime-silica glasses whose compositions are indicated in Fig. 1. The compositions of his glasses were calculated from the batches melted, supplemented by four analyses showing impurities introduced by attack on the clay pot in which they were melted. The matter of annealing was not mentioned. His densities and refractive indices differ from our curves, except in one case, less than might be expected from differences in annealing.

The density of a series of soda-lime-silica glasses was measured by English and Turner,[37] and the optical properties of the same glasses by Clarke and Turner.[38] The compositions of the glasses as determined by analysis are given by English and Turner[39] and by Davidson and Turner,[40] and are indicated in Fig. 1. Both density and refractive index were determined on annealed glasses, but no details of the treatment are given. The "annealing temperatures" of these same glasses had previously been determined by English and Turner,[41] and ranged from 490° for a glass containing but 0.2 percent of CaO, to 601° for the glass richest in CaO. After correcting for the impurities as indicated above, their results agree with our curves within the amounts to be expected from the differences in annealing.

The work of Faick and Finn was mentioned under the soda-silica glasses. In addition they determined the refractive index of 21 soda-lime-silica glasses, covering a part of the lower portion of our diagram. The compositions of their glasses are indicated in Fig. 1. Their general results on refractive index of glasses annealed "at high temperatures" are in excellent agreement with the refractive indices of our unannealed

[36] C. J. Peddle, J. Soc. Glass Tech., *4*, p. 20; 1920.
[37] S. English and W. E. S. Turner, J. Soc. Glass Tech., *4*, p. 126; 1920.
[38] J. R. Clarke and W. E. S. Turner, J. Soc. Glass Tech., *4*, p. 111; 1926.
[39] S. English and W. E. S. Turner, J. Soc. Glass Tech., *3*, p. 125; 1919.
[40] J. H. Davidson and W. E. S. Turner, J. Soc. Glass Tech., *3*, p. 222; 1919.
[41] S. English and W. E. S. Turner, J. Soc. Glass Tech., *3*, pp. 125, 278; 1919.

glasses, that is, their values are 0 to 0.0025 (average 0.001) lower than ours for annealed glasses. Some curves showing the densities of glasses covering the same composition range have been published,[42] but the complete data suitable for comparison have not yet appeared.

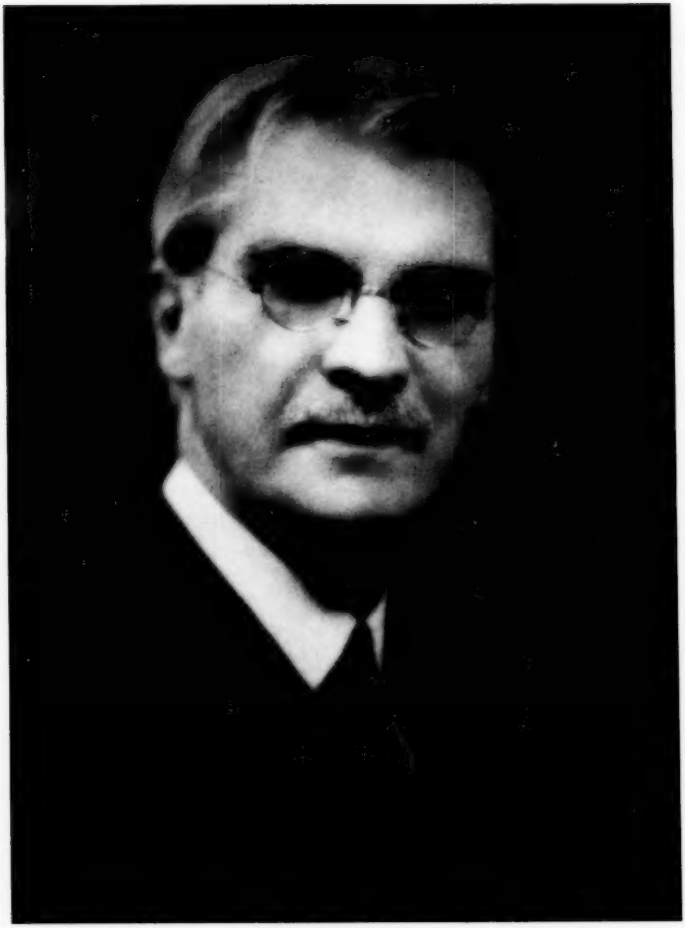
Gehlhoff and Thomas[43] measured the density of strained, fire-polished glasses derived by alteration of composition along a single line in the ternary system, $Na_2O-CaO-SiO_2$. Detailed results are not given, nor were the glasses analyzed, so a comparison of results would be devoid of interest.

SUMMARY

Measurements of density and optical properties have been made on 185 glasses of known composition in the ternary system, $Na_2O-CaO-SiO_2$, well distributed over the composition range from mixtures intermediate between the orthosilicate and metasilicate ratios to pure silica. The measurements include both unannealed and annealed glasses, and a special study was made of the effect of heat treatment on the properties of the glasses. Optical measurements include the refractive index, n_D, dispersion, n_F-n_C, and in addition measurements of partial dispersion on glasses of extreme types. The relations between composition and properties are discussed, and it is concluded that no justification exists for an assertion that definite compounds exist in, and determine the properties of, the glasses. The specific refractivity as calculated on the hypothesis of additivity has been calculated by means of the Gladstone-Dale, Lorentz-Lorenz, Eykman, and Lichtnecker formulae, and the same general pattern of percentage departure was found for all the formulae tested. The Gladstone-Dale formula gives percentage departures that are little more than one-half as large as the Lorentz-Lorenz or Eykman formulae, and one-third as large as the Lichtnecker formula.

[42] Ceramic Industry, p. 92; Feb. 1932.

[43] G. Gehlhoff and M. Thomas, Zeits. f. tech. Physik, 7, p. 105; 1926.



Adolph Lomb

In Memoriam

ADOLPH LOMB
(15 July, 1866–30 September, 1932)

For the Optical Society of America the year 1932 has been a sorrowful year on account of the loss by death of several of its most eminent and beloved members, chief among whom may be named:

CHARLES SHELDON HASTINGS (d. 31 January, 1932; *aet.* 83), emeritus-professor of physics in Yale University, who was a pioneer in applied optics in this country and who, together with the late Professor MICHELSON, was one of the small number of Honorary Members of the Optical Society; LEONARD THOMPSON TROLAND (d. 27 May, 1932; *aet.* 43), gifted, versatile and handsome, who was president of the Optical Society in 1922–3; IRWIN GILLESPIE PRIEST (d. 19 July, 1932; *aet.* 46), clever and original as an investigator with a keen scientific mind, who was president of the Optical Society in 1928–29; and last but not least, ADOLPH LOMB (d. 30 September, 1932; *aet.* 66), vice-president of the Bausch and Lomb Optical Company, beloved and honored treasurer of the Optical Society from the time it was founded sixteen years ago until the day of his death.

Mr. Lomb was a captain of industry in the highest sense of the word. Highly educated and cultivated, a student of men and affairs, he was alive to all that was going on in the complex modern world and took the keenest interest in everything that concerned the welfare of mankind. Broad and noble in his sympathies, thoughtful and wise in his judgments, and generous by instinct, he was a humanitarian who could have boasted (except that he never boasted) that nothing human was alien to him. Science, civilisation, progress—to Adolph Lomb these all meant the same thing; for he believed passionately that the material and spiritual betterment of the human race was dependent on the advancement of knowledge, and that science, like virtue, "is its own exceeding great reward."

Modest, reserved, almost shy, he lived his days simply and quietly, punctually attending to his business and faithfully performing the task that was appointed to be done. He did not actively participate in public affairs nor was his name often seen in print, yet he was connected with enterprises of great pith and moment, and deeply concerned in all the big questions of the day, ever hopeful that right would prevail. The memory of his father. who had occupied a post of high honor and esteem in the community where he lived, he revered with filial piety, and he strove always to follow in his footsteps. Between him and his mother a charming, cultivated lady, now nearly ninety years of age in the full possession of all her faculties, there existed a relationship of tender love and mutual sympathy which it was a joy and a privilege for any outsider to behold. Of late years Mr. Lomb's health, never very robust,

had begun to fail visibly more and more, to the great anxiety and fear of all his friends and associates, but he bore these troubles with uncomplaining fortitude. Yet his death which occurred on the last day of September and was brought on by pneumonia came rather suddenly and unexpectedly. At least neither he nor his devoted mother and brother who watched by his bedside had to endure the agony and grief of a long-protracted illness.

Adolph Lomb was never married. If he had one love in the world, it was for the Optical Society of America which he had helped to organise and which he always continued to support. No infirmity of strength or impairment of eyesight, no long railway journey (which he always dreaded) could keep him from attending punctually each of its appointed meetings. Every member of the Society knew him by sight and greeted him cordially in the remote corner where he usually found his seat; but not many of them perhaps knew that on more than one occasion when the affairs of the Society were in a somewhat critical situation this quiet, unassuming gentleman had been a tower of strength and that but for his prompt and effectual aid the Optical Society might have found it difficult to carry on its work. It was entirely owing to Mr. Lomb that the Optical Society was enabled to publish the English translation of Helmholtz's treatise on physiological optics.

He died without making a will, and so he left no bequests. A man of large means, he had enjoyed the luxury and the privilege of giving all his life long. Nobody, not even his most intimate friends, will ever know the extent of his benefactions which were very widespread and generous. He never alluded to them and tried to hide them as much as possible. Entirely free from ostentation, he was always eager to help where help was needed, yet not easily persuaded to give until he had thoroughly investigated the cause and found it high and worthy. Others too give money and, one might say, give offense. Adolph Lomb gave liberally not only money but time and pains and sympathetic consideration, and was ready to give again if the occasion demanded it.

The preliminary report of the Treasurer of the Optical Society, prepared for the Lake Placid Meeting in the middle of October, was dated 30 September, the day he died. The last cheque he signed in his official capacity, perhaps the last deed he performed on earth, was executed on his death-bed with the aid of his brother who held his hand and guided the faltering pen.

"His life was gentle; and the elements
So mix'd in him, that Nature might stand up
And say to all the world: This was a man."

COLUMBIA UNIVERSITY,
NEW YORK, N.Y.
25 October, 1932

J.P.C.S.